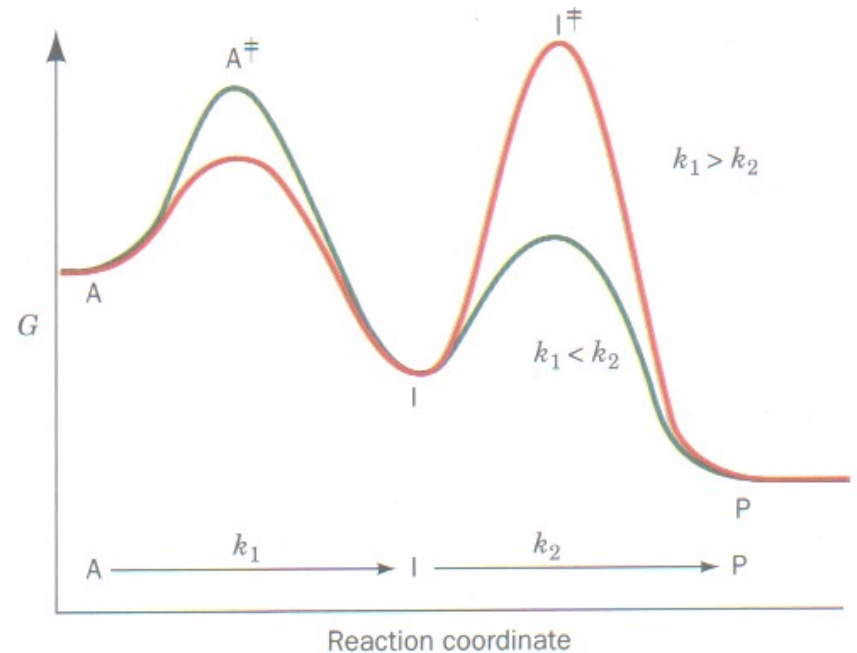


# Chapter 14: Rates of Enzymatic Reactions

Voet & Voet:  
Pages 472-493





# Initial Velocity

**Steady State assumption: Allows reaction rate equation (in term of [ES]) to be rewritten in terms that are experimentally measurable**

$$0 = d \frac{[ES]}{dt} = k_1[E_T - E][S] - k_{-1}[ES] - k_2[ES]$$

**reaction rate is generally measured for a short duration after reaction start (initial velocity,  $v_o$ )**

$$v = d \frac{[P]}{dt} = k_2[ES] = k_2 \frac{([E_T][S])}{(K_M + [S])}$$

**ie. before significant amounts of substrate (10%) have been converted to product**

**Minimizes possible back reactions, product inhibition and other phenomenon**

**The maximum velocity of the enzyme catalyzed reaction occurs when  $[E_{total}] = [ES]$**

$$V_{max} = k_2[E_T]$$

**Result is the Michaelis-Menten equation; the basic equation of enzymology**

$$v_o = \frac{(V_{max}[S])}{(K_M + [S])}$$



# $K_M$ – Michaelis constant

Plots of initial velocity vs. substrate concentration yield hyperbolic curves for Michaelis-Menten reactions

$K_M$  is the substrate concentration at which the reaction velocity is  $\frac{1}{2}$  maximal

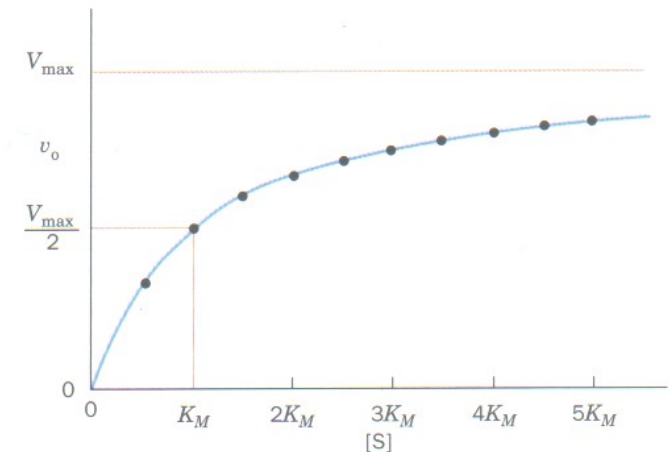
Proof: when  $K_M = [S]$ , the Michaelis-Menten equation reduces to  $v_o = V_{max}/2$

small  $K_M$  is associated with efficient catalysis at low  $[S]$

For  $k_2 \ll k_{-1}$ ,  $K_M$  is a measure of substrate affinity

Restatement of equilibrium assumption

$$v_o = \frac{V_{max}[S]}{K_M + [S]}$$





# Analysis of Kinetic Data

Several methods exist for determining the kinetic constants in the Michaelis-Menten equation

(1)  $v_o$  vs  $[S]$  plots require extrapolation of  $V_{max}$

(2) Lineweaver-Burk (double reciprocal) plot:  $1/v_o$  vs  $1/[S]$

Better estimate of  $V_{max}$  ... but most measurements are near the y axis and small errors can skew calculation of constants

(3) Eadie-Hofstee (double reciprocal  $\times V_{max}$ ) plot:  $v_o$  vs  $v_o/[S]$

Improved version of Lineweaver-Burk that is less sensitive to small errors

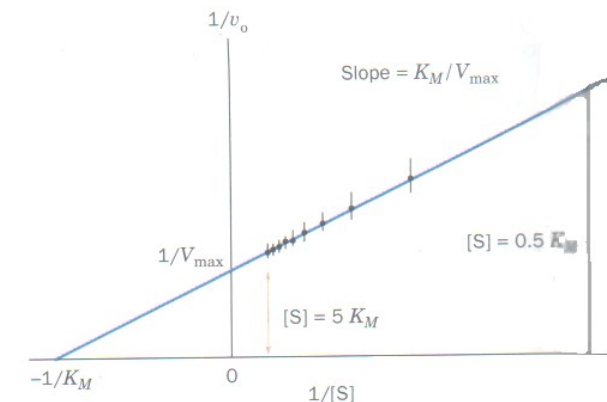
(4) Regression methods and initial velocity plots (Best)

Computational methods overcome extrapolation problem

$$v_o = \frac{V_{max} [S]}{K_M + [S]}$$

$$\frac{1}{v_o} = \frac{K_M + [S]}{V_{max} [S]}$$

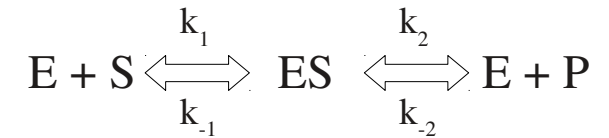
$$\frac{1}{v_o} = \frac{K_M}{V_{max}} \frac{1}{[S]} + \frac{1}{V_{max}}$$



# Modified Michaelis Menten

Michaelis-Menten equation can be modified to consider the back reaction from product to substrate

This treats [ES] as a reaction intermediate and may be considered a one intermediate Michaelis-Menten equation



Much more complicated but still contains terms that are measurable and solvable

$V_{f,max}$  and  $V_{r,max}$  are the maximal velocities of the forward and backward reaction

$K_{f,s}$  and  $K_{r,p}$  are the Michaelis constants for the forward and backward reaction

$$v = \frac{\left( \frac{(V_{(f,max)}[S])}{K_{(s,M)}} - \frac{(V_{(r,max)}[P])}{K_{(p,M)}} \right)}{\left( 1 + \frac{[S]}{K_{(s,M)}} + \frac{[P]}{K_{(p,M)}} \right)}$$

*Not responsible for deriving or reproducing equation but must recognize various symbols*

# Haldane Relationship

**For modified Michaelis-Menten equations that consider back reactions, the Haldane Relationship can be utilized to determine  $K_{eq}$  and (sometimes) other kinetic constants**

**at equilibria,  $v = 0$  and the rates of formation and breakdown of product are constant**

**modified Michaelis-Menten equation is still valid at equilibria ( $v = 0$ ) and can be reduced to the Haldane Relationship**

***Haldane Relation shows the kinetic parameters for an enzyme catalyzed reversible reaction are related by the equilibrium constant***

$$K_{eq} = \frac{[P]_{eq}}{[S]_{eq}} = \frac{(V_{(max, f)} K_{(M, p)})}{(V_{(max, r)} K_{(M, s)})}$$

# Steady State Limitations

**Steady-State (modified Michaelis-Menten) Kinetic Analysis of a reaction cannot unambiguously establish its mechanism**

**Can experimentally determine up to 4 kinetic parameters ( $V_{\max}$  and  $K_M$  for forward and reverse reaction)**

**Modified Michaelis-Menten equations with 2 or more reaction intermediates have the same form (but more kinetic parameters) as the one intermediate case**

**Cannot determine these additional terms using steady state kinetic approaches as we only have 4 known quantities**

**In general, Steady State approaches can only monitor reaction steps that consume or produce the substrate or product**

**Kinetic data not compatible with a given mechanism can be used to reject that mechanism**

# Inhibition

**Inhibitors are compounds that bind to enzyme in such a way that the activity of the enzyme is reduced**

- (1) many inhibitors are substances that resemble the enzyme's natural substrate but react slowly or not at all**
- (2) inhibitors are commonly used to investigate the nature of the substrate-binding site and to elucidate enzyme mechanism**
- (3) Most clinical drugs are enzyme inhibitors produced synthetically or isolated from natural sources**

**recently the first rationally 'designed' drugs have been utilized**

**Large portion of pharmacology is medicinal use of inhibitors**

**must consider many issues over and above binding**

**dosage, solubility, side effects, cost, *etc***



# Example: methotrexate

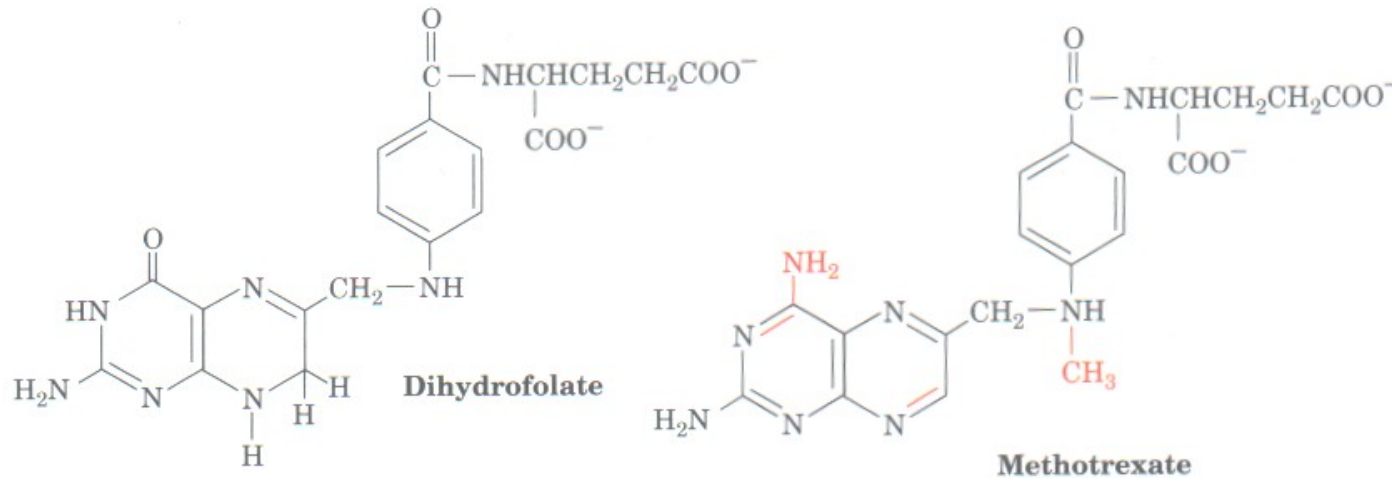
## Enzyme inhibitor used as chemotherapeutic agent

dihydrofolate reductase catalyzes an essential step in the synthesis of dTMP

methotrexate binds to and inhibits dihydrofolate reductase preventing dTMP biosynthesis

preferentially kills actively dividing cells (eg cancer cells)

Methotrexate is structurally related to the normal substrate and binds in the enzyme active site in a conformation similar but not identical to the substrate

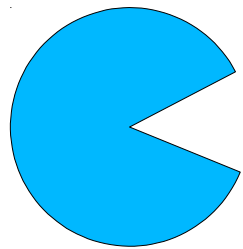


# Types of Inhibition

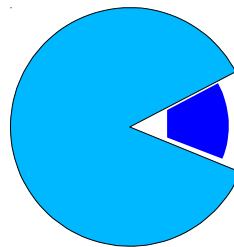
Enzyme inhibitors have a variety of mechanisms

Consider 3 simple inhibitor mechanisms

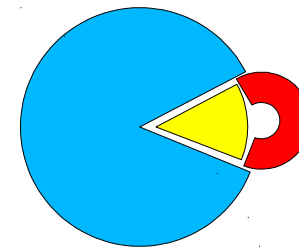
- (1) Competitive inhibition – inhibitor competes with normal substrate for enzymatic binding site
- (2) Uncompetitive inhibition – inhibitor only binds to the enzyme substrate complex
- (3) Mixed inhibition – inhibitor binds both to the substrate binding site of the enzyme and to the enzyme substrate complex (both competitive and uncompetitive)



substrate



competitive  
inhibitor



uncompetitive  
inhibitor

# Competitive Inhibition

(1) **Competitive inhibition** – inhibitor competes with normal substrate for enzymatic binding site

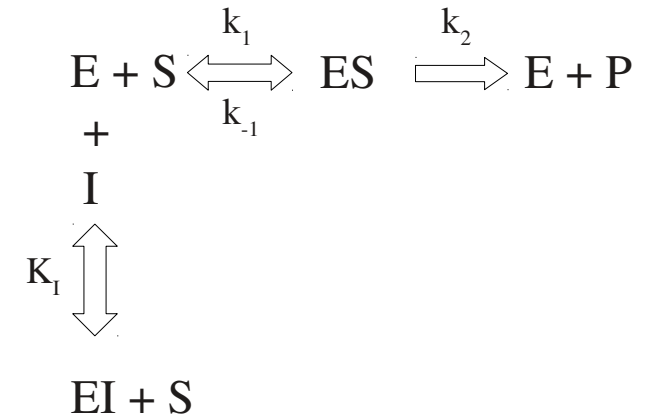
Inhibitor structurally resembles the substrate

**General chemical reaction for competitive inhibition**

assumes inhibitor (I) binds reversibly and is in rapid equilibrium so that  $K_i = [E][I]/[EI]$

assumes EI is inactive (no reaction)

***Competitive inhibitors reduce the [E] available for substrate binding***



# Competitive Inhibition equation

Michaelis-Menten equation must reflect

$$E_T = E + ES + EI$$

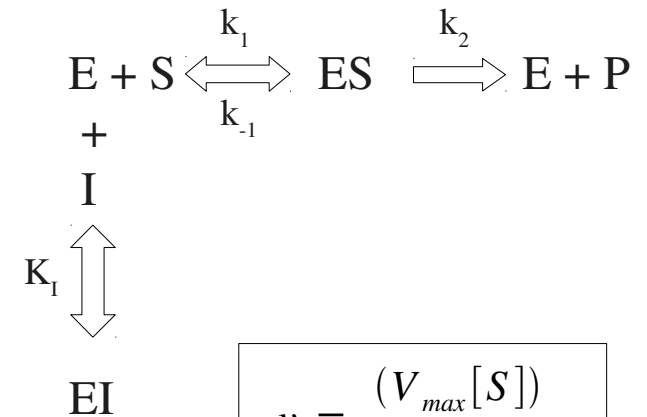
Derivation requires developing expressions for [E] and [EI]

Ultimately yields a Michaelis-Menten equation with the  $K_M$  modulated by a factor,  $\alpha$

$\alpha$ , depends upon the ratio of [I] and  $K_I$

inhibition affects  $K_M$  by reducing the concentration of the enzyme [E]

inhibition does not affect  $V_{max}$  as it can be reversed a higher substrate concentrations



$$v_o = \frac{(V_{max} [S])}{(\alpha K_M + [S])}$$

$$\alpha = \left(1 + \frac{[I]}{K_I}\right)$$

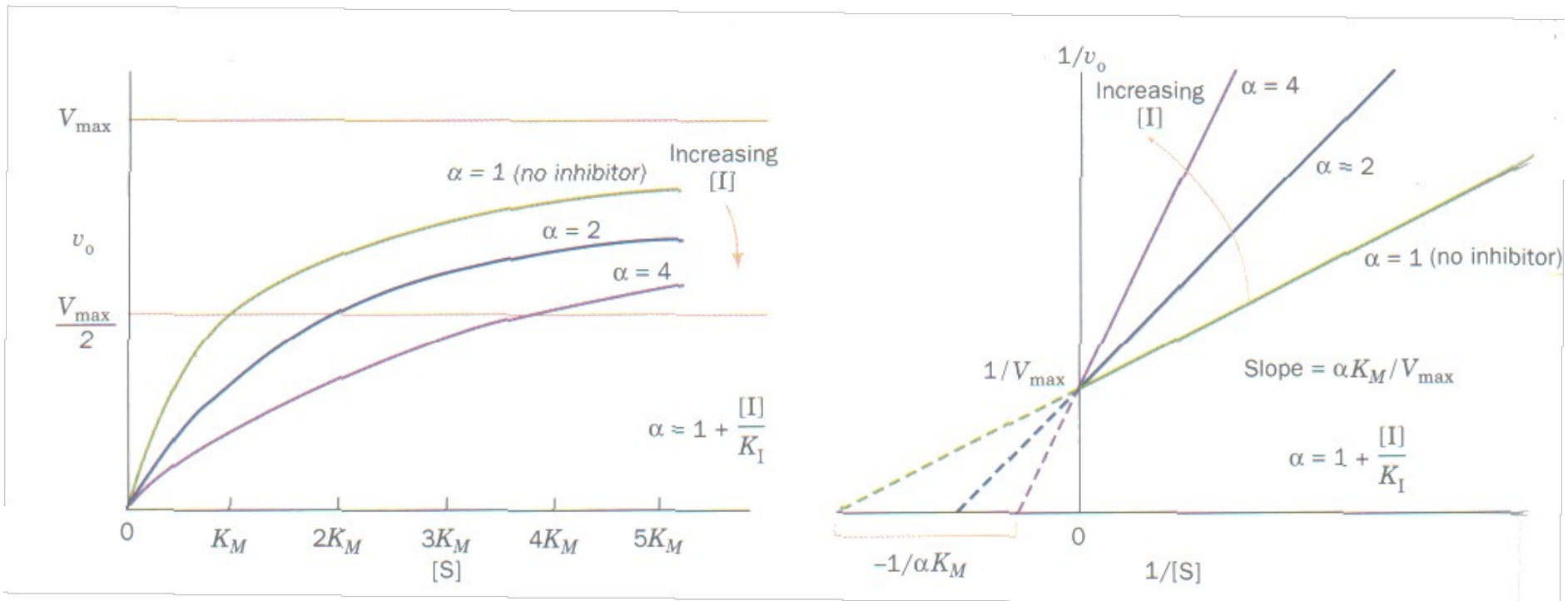
# Competitive Inhibition plot

## Initial reaction rates and double-reciprocal plots for a reaction in the presence of varying [I]

increases the  $K_M$  and causes  $K_M$  dependent parameters (slope and x-intercept) to vary accordingly

does not affect  $V_{max}$  dependent y-intercept

$$\frac{1}{v_o} = \left(\alpha \frac{K_M}{V_{max}}\right) \left(\frac{1}{[S]}\right) + \frac{1}{V_{max}}$$





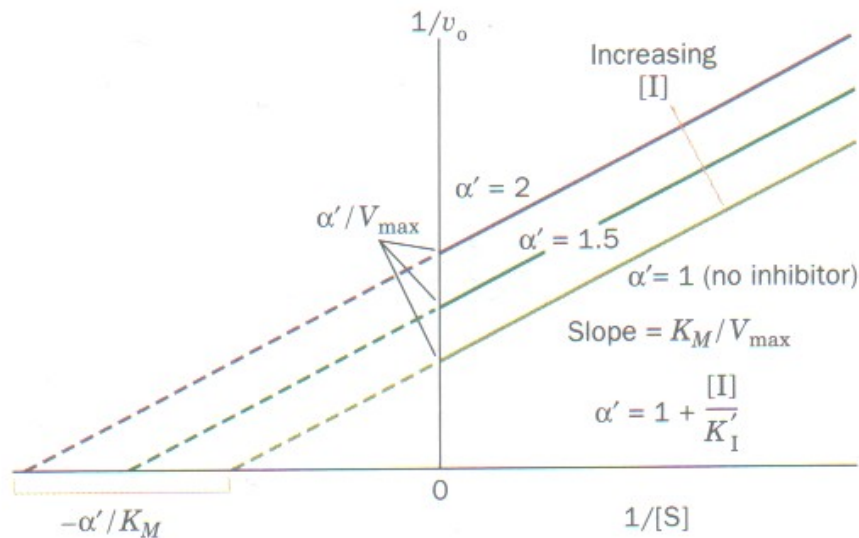
# Uncompetitive Inhibition plot

Double-reciprocal plots for a reaction in the presence of varying [I]

affects intercept terms

does not affect slope term ( $K_M / V_{max}$ )

$$\frac{1}{v_o} = \left(\frac{K_M}{V_{max}}\right)\left(\frac{1}{[S]}\right) + \frac{(\alpha')}{V_{max}}$$



# Mixed Inhibition

## (3) Mixed inhibition – inhibitor binds to the free enzyme and the enzyme-substrate complex

Mixed inhibitors bind sites that participate in both substrate binding and catalysis

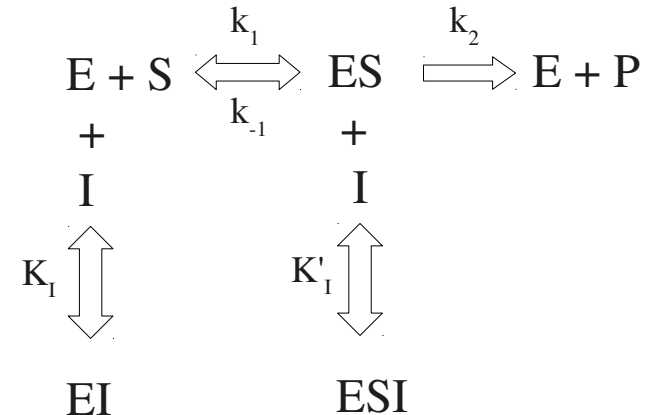
### General chemical reaction for mixed inhibition

assumes inhibitor binding is in equilibrium and each step has its own dissociation constant

$$K_I = [E][I]/[EI]$$

$$K'_I = [ES][I]/[ESI]$$

Mixed inhibitors have properties of both competitive and uncompetitive inhibitors



$$v_o = \frac{V_{max}[S]}{(\alpha K_M + \alpha'[S])}$$

$$\alpha = \left(1 + \frac{[I]}{K_I}\right)$$

$$\alpha' = \left(1 + \frac{[I]}{K'_I}\right)$$

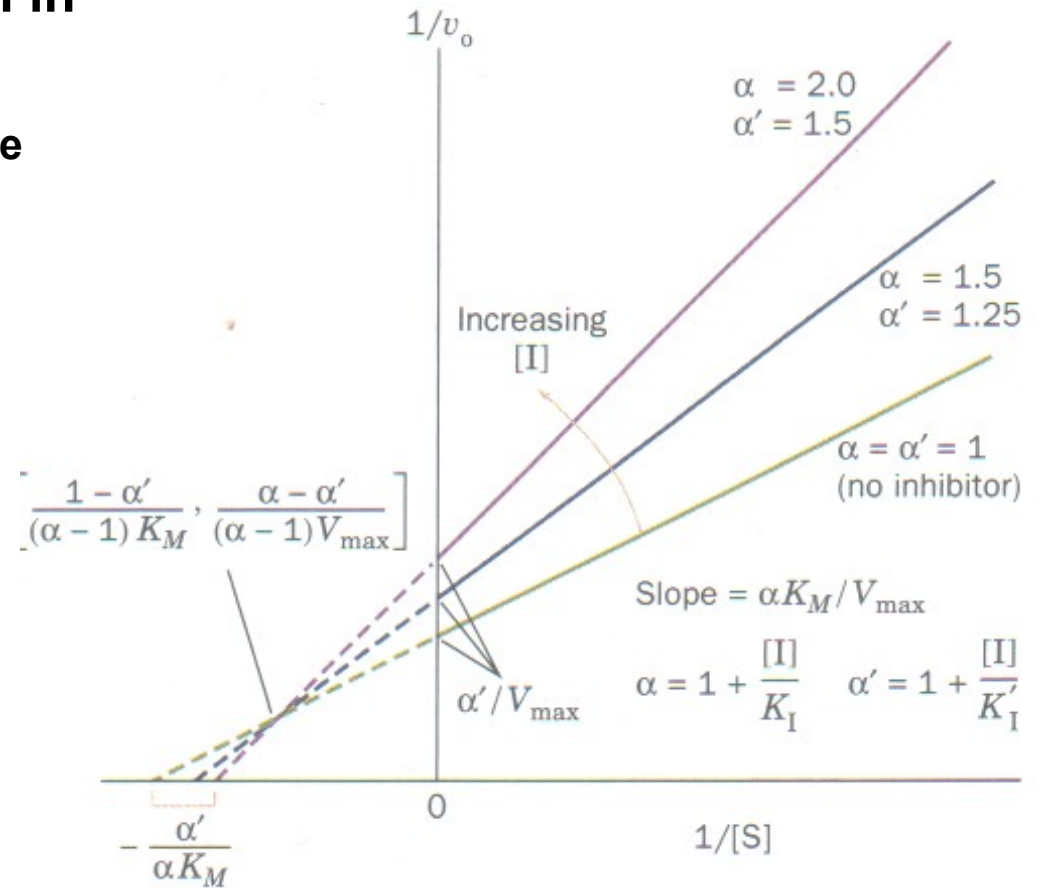


# Mixed Inhibition Plots

## Double-reciprocal plots for a reaction in the presence of varying [I]

affects both intercepts like uncompetitive  
affects slope like competitive

$$\frac{1}{v_o} = \left(\frac{\alpha K_M}{V_{max}}\right) \left(\frac{1}{[S]}\right) + \frac{(\alpha')}{V_{max}}$$





# Summary of Inhibition

The values of  $K_M$  and  $V_{\max}$  measured in the presence of inhibitor are referred to as the apparent  $K_M$  ( $K_M^{\text{app}}$ ) and the apparent  $V_{\max}$  ( $V_{\max}^{\text{app}}$ )

This would represent the case where you have not accounted for the inhibitor in your initial rate or double-reciprocal plot

The effects of inhibitors on  $K_M^{\text{app}}$  and the apparent  $V_{\max}^{\text{app}}$  can be used to identify the type of inhibition

Type of Inhibition	$V_{\max}^{\text{app}}$	$K_M^{\text{app}}$
None	$V_{\max}$	$K_M$
Competitive	$V_{\max}$	$\alpha K_M$
Uncompetitive	$V_{\max}/\alpha'$	$K_M/\alpha'$
Mixed	$V_{\max}/\alpha'$	$\alpha K_M/\alpha'$

$$^{\alpha}\alpha = 1 + \frac{[I]}{K_I} \text{ and } \alpha' = 1 + \frac{[I]}{K'_I}$$

# Effects of pH

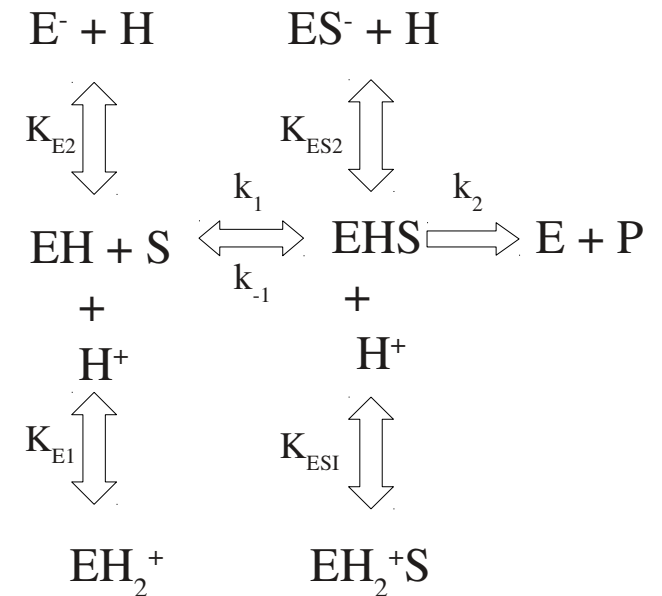
Enzymes are pH sensitive and are usually only active in a narrow pH range (usually 5-9) due to:

- (1) pH sensitivity of substrate binding
- (2) reduced catalytic efficiency of the enzyme
- (3) ionization of substrate
- (4) protein structural changes (usually at pH extremes)

Kinetic analysis as a function of pH provides information about the nature of functionally (catalytically) important residues

general chemical equation for enzyme with two ionizable groups

assumes only EH and ESH are active

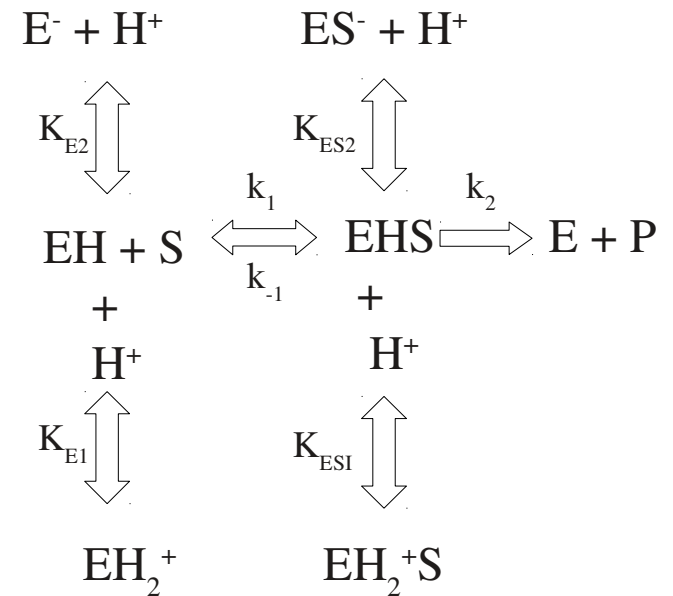
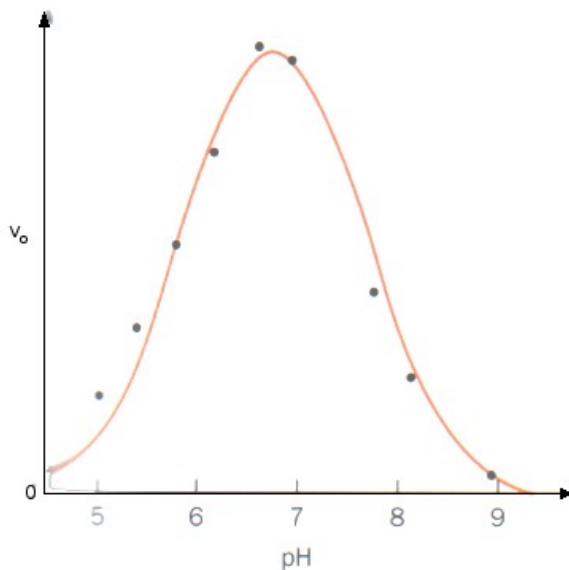


# Kinetic equations and pH

At each pH, the enzyme behaves according to the simple Michaelis-Menten equation

the pH dependence of  $f_1$  and  $f_2$  causes  $v_o$  to vary in a bell-shaped manner

$f_1$  and  $f_2$  modulate the amount of EH and EHS at a given pH



$$v_o = \frac{(V_{max}'[S])}{(K_M' + [S])}$$

$$K_M' = K_M \left( \frac{f_1}{f_2} \right) \quad V_{max}' = \frac{V_{max}}{f_2}$$

$$f_1 = \frac{[H^{+1}]}{K_{E1}} + 1 + \frac{K_{E2}}{[H^{+1}]}$$

$$f_2 = \frac{[H^{+1}]}{K_{ES1}} + 1 + \frac{K_{ES2}}{[H^{+1}]}$$

# Evaluation of Ionization constants

The ionization constants in our pH dependent general chemical reaction can be determined by plotting  $\log V'_{\max}$  vs pH and  $\log (V'_{\max} / K'_M)$  vs pH

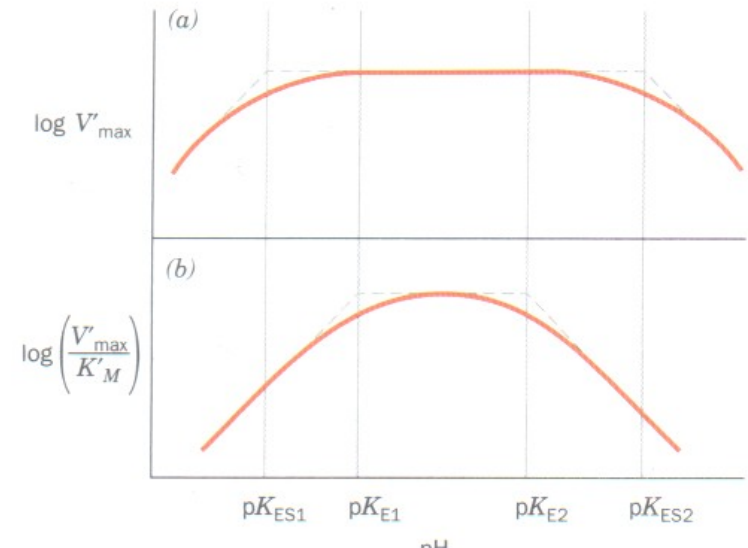
$\log V'_{\max}$  vs pH provides information about  $K_{ES1}$  and  $K_{ES2}$  or the final step of the reaction

$\log (V'_{\max} / K'_M)$  vs pH provides information about  $K_{E1}$  and  $K_{E2}$  or the first step of the reaction

Knowing the ionization constant of a catalytic residue gives a clue to its identity

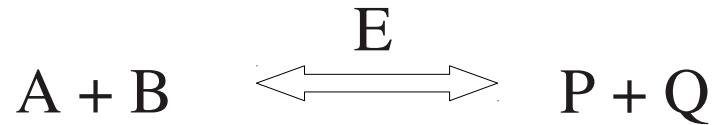
a  $pK_{E1}$  of 4 would suggest a carboxylate group such as an Asp or Glu contains

the experimentally determined pKs may deviate considerably from the  $pK_a$  of equivalent free amino acids due to the local environment



# Bisubstrate Reactions

~60 % of all enzymatic reaction are bisubstrate reactions that require two substrates and yield two products



Almost all bisubstrate reactions fall into two groups:

***transferase reactions*** – one functional group is transferred between the substrates

***redox reactions*** – reducing equivalents are transferred between the substrates

While there are many possible bisubstrate reaction mechanism, only a few types commonly occur



# Reaction Terminology

## Clelland Nomenclature

**Substrates** are A, B, C, D *in the order they add to the enzyme*

**Products** are P, Q, R, S, T *in the order they leave the enzyme*

**Stable enzyme forms** are E, F, G

stable enzyme forms are defined as those incapable of converting to another stable form by itself

Number of reactants and products in a given reaction are specified by the terms *uni*, *bi*, *ter* and *quad*

eg. Reaction with 1 substrate and two products - Uni Bi reaction

Bisubstrate reactions with two products - Bi Bi reaction

# Bisubstrate Reaction Types

Two major mechanism for bisubstrate reactions

**(1) Sequential Reactions** – all substrates must combine with the enzyme before the reaction can occur and products can be released

Single step conversion of substrate to product gives rise to alternate name, single displacement reaction

Subdivided into Ordered and Random

**Ordered Sequential** mechanism require each substrate to bind and each product to be released in particular order

**Random Sequential** mechanism allow substrate to bind and product to be released in any order

**(2) Ping-Pong Reactions** – one or more products are released before all substrates have combined with the enzyme

also referred to as double displacement reaction



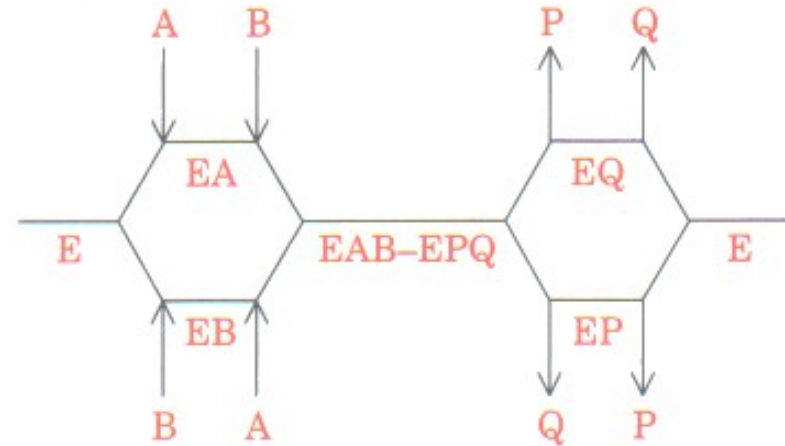
# Bisubstrate Reaction Types

Shorthand notation for various  
reaction types (Clelland)

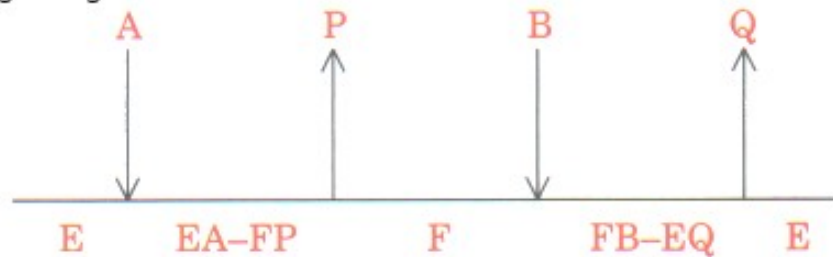
Ordered Sequential Bi Bi



Random Sequential Bi Bi



Ping Pong Bi Bi



# Bisubstrate Reaction Rates

As one would expect, the various bisubstrate mechanism can be distinguished using initial rates kinetics

somewhat more complex for bisubstrate than unisubstrate

Ordered Bi Bi Reaction rate equation

Ping-Pong Reaction rate equation

$$\frac{1}{v_o} = \frac{1}{V_{max}} + \frac{K_M^A}{(V_{max}[A])} + \frac{K_M^B}{(V_{max}[B])} + \frac{(K_M^A K_M^B)}{(V_{max}[A][B])}$$

$$\frac{1}{v_o} = \frac{K_M^A}{(V_{max}[A])} + \frac{K_M^B}{(V_{max}[B])} + \frac{1}{V_{max}}$$

where  $K_M^A$  and  $K_M^B$  are the [A] and [B] required for half maximal activity.

$V_{max}$  is maximal velocity when both A and B are saturating