

Chemistry 5850 Summer 2004 Assignment 10

Due: Thursday, July 22

Weight of this assignment: 28 marks

In this assignment, you will analyze the cusp map:

$$x_{n+1} = 1 - a\sqrt{|x_n|}.$$

The parameter a will be taken to be positive, but the iterates x_n can be either positive or negative. In the questions which follow, feel free to use appropriate software if it facilitates your computations.

1. Sketch the graph of x_{n+1} vs x_n . Where have you seen a map that looks like this before? [3 marks]
2. Find all fixed points of this map and analyze their stability. [10 marks]
3. Plot the second power of the map for a few different values of a . Is there always a period-2 orbit (other than the fixed points of the cusp map itself)? [5 marks]
Hint: You can find fixed points graphically by finding intersections of the map with the line $x_{n+1} = x_n$.
4. Plot the Liapunov exponent as a function of a . Annotate your graph, identifying any major features you see (e.g. chaotic regions, periodic windows). [10 marks]