Chemistry 5850 Fall 2005 Assignment 9

Due: Monday, Nov. 21.

Weight of this assignment: 36 marks

1. In the implicit Euler method, we approximate the solution to a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ by solving the equation

$$\frac{\mathbf{x}_{n+1} - \mathbf{x}_n}{h} = \mathbf{f}(\mathbf{x}_{n+1})$$

for $\mathbf{x}_{n+1}$, where $\mathbf{x}_n$ and $\mathbf{x}_{n+1}$ are two values of $\mathbf{x}$ separated in time by h. In special cases, we can write an explicit map. This is the case for instance for the logistic differential equation $\dot{x} = x(1-x)$. Write down this map. Does it suffer from the same problems as the explicit Euler method applied to this problem for larger values of h? [6 marks]

2. Poincaré maps can be used to study the approach of a system to its attractor. This is a particularly effective technique when the attractor is a simple one, e.g. a limit cycle.

   (a) Explain what behavior you would expect to see in the Poincaré map of a system with a stable limit cycle. [4 marks]

   (b) If possible, derive analytically the Poincaré map of the differential equation $\dot{x} = -x + \cos t$ studied in lecture 8. Show that this map behaves as expected. If you can't get an analytic map, you should be able to at least write down an implicit map and then use Maple to calculate iterates. [6 marks]

3. Study the following system thoroughly using Poincaré map techniques. [20 marks]

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -az - y + |x| - 1. \end{aligned}$$