Time scales and solution domains in singularly perturbed systems

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Initial conditions of singularly perturbed equations

- Suppose that we have a singularly perturbed system

$$\frac{dx}{dt} = f(x, y)$$
$$\epsilon\frac{dy}{dt} = g(x, y)$$

  with general initial conditions $(x, y) = (x_0, y_0)$.
- Tikhonov's theorem essentially says that, if ϵ is sufficiently small, the solutions of this system will approach those of

$$\frac{dx}{dt} = f(x, y)$$
$$g(x, y) = 0$$

- However, the initial conditions of the degenerate system cannot be the same as the initial conditions of the original system since, in general, $g(x_0, y_0) \neq 0$.

Two time scales implied

$$\frac{dx}{dt} = f(x, y)$$
$$\epsilon\frac{dy}{dt} = g(x, y)$$

- A singularly perturbed system implied two distinct time scales:
  1. A time scale of $O(\epsilon)$ over which the solutions approach the curve $g(x, y) = 0$.
     $\Longrightarrow$ inner solution
  2. A time scale of $O(1)$ over which the solutions evolve along $g(x, y)$.
     $\Longrightarrow$ outer solution

Solution structure