Practice exercises for test 2

Marc R. Roussel
Department of Chemistry and Biochemistry
University of Lethbridge

October 27, 2019

There is **no assignment** this week so you can focus on studying for the test.

The following problems practice the skills you will need to complete the test, which will cover sections 4.5, 5.3, and chapter 6. Note that many of the skills covered previously are still needed to solve problems in these sections.

1. Problems 4.3 and 4.4 from the textbook

2. Problems 6.1 and 6.3–6.6 from the textbook. Note that several of these are open-ended. There may be a problem like this on the test, which will allow you to show off what you know how to do (and, by implication, to avoid things you're less good at). If I do put a question like this on the test, I will establish a set of possible tasks to be completed, along with point values for each task. For example, I might allow 4 marks for finding equilibrium points, 5 marks for analyzing the stability of each equilibrium point, etc. (The exact number of marks will depend on the complexity of the work to be completed. For example, I wouldn't allocate as many marks for finding equilibria of a system with a single equilibrium at $(0, 0)$ as I would for a system with, say, three equilibrium points.) The total number of marks available will exceed the value of the question. However, there will be no extra credit if you (correctly) complete work worth more than the value of the question. This means that you will want to strategize about what you want to take on given the time available to you.

3. Consider the rate equations

$$\frac{dx}{dt} = k_1 x^2 y - k_2 x^5,$$
$$\frac{dy}{dt} = -k_3 y + k_4 x^2,$$

where all of the constants k_i are assumed to be positive. Note: to my knowledge, this isn't a model of anything in particular. It's a set of equations cooked up by Wiggins[1] to illustrate a particular point.

Work out the stability of the equilibrium point using centre-manifold theory.

[1] *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Springer: New York, 1990, p. 196.