Chemistry 4010 Fall 2019 Assignment 2

Due: Sept. 24, 6:00 p.m.

Total marks: 38 marks

Unless otherwise specified there is no need to reduce equations to dimensionless form in this assignment.

1. Carry out a linear stability analysis of the mass-spring system in example 3.3 of the textbook. **5 marks**

2. For the mechanism **20 marks**

$$\mathrm{A} \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2\,\mathrm{X}, \tag{1}$$

$$\mathrm{X} \xrightarrow{k_2} \mathrm{P} \tag{2}$$

   (a) Obtain the mass-action rate equations and reduce them to a pair of dimensionless differential equations. [10 marks]

   (b) Carry out a linear stability analysis. [10 marks]

3. Some systems have equations that are most conveniently expressed in polar form. Consider for example the following system: **13 marks**

$$\frac{d\theta}{dt} = \omega,$$
$$\frac{dr}{dt} = -ar, \qquad \text{defined on } r \geq 0 \text{ with } a > 0.$$

   (a) What happens if you try to apply linear stability analysis to this problem without taking into account the polar representation, i.e. by treating θ and r as ordinary variables? [2 marks]

   (b) Since r represents the distance from the equilibrium point and the two equations are not coupled, you can in fact just analyze the second equation by itself. Apply linear stability analysis to this equation. [5 marks]

   Hint: If c is a number, $\det(c) = c$. Don't be fooled by the notation $|c| = \det(c)$. In this context, the bars **do not** represent absolute values.

   (c) Verify that $V(r, \theta) = r^2$ is a Lyapunov function for this system. What does the existence of a Lyapunov function prove? [6 marks]