Chemistry 4010 Fall 2019 Assignment 1

Due: Sept. 17, 6:00 p.m.

Total marks: 35

Note: Neatness counts! I don't mind handwritten assignments, but what you present should be readable and easy to follow.

1. The logistic differential equation is an important model for the time evolution of natural populations in an environment where resources constrain growth: **11 marks**

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

In this equation, P typically represents the population density (number of individuals per unit area), K is called the carrying capacity, and r is called the specific growth rate.

   (a) Reduce this equation to dimensionless form. [4 marks]

   (b) Carry out an analysis of the flow on the line implied by this equation (analogous to a phase-plane analysis, except in one dimension). Consider both positive and negative values of P. Show all steps, but display the final results of your analysis graphically. [5 marks]

   (c) Given any initial $P > 0$, what happens to the population at large t? Why do you think that K is called the carrying capacity? [2 marks]

2. In this problem, you will consider the Michaelis-Menten mechanism **24 marks**

$$\mathrm{E} + \mathrm{S} \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} \mathrm{C} \xrightarrow{k_{-2}} \mathrm{E} + \mathrm{P}$$

   (a) Write down the mass-action equations, and reduce them to a pair of coupled dimensionless differential equations. [10 marks]

   Hint: Show that [E] + [C] is a constant, and use that to eliminate [E].

(b) Sketch the resulting flow in the phase plane. As usual, show your work, but make sure to include a final graphical result. [10 marks]

(c) Confirm your hand-drawn sketch with an XPPAUT calculation showing the nullclines and flow. [4 marks]

Note: You can just print out your code and figure and attach them to your assignment. I would recommend that you use a screen grab utility to get the graphic from XPPAUT.