Chemistry 2740 Spring 2020 Test 3 Solutions

1. (a) $\dfrac{d[\mathrm{C}]}{dt} = k_{\mathrm{on}}[\mathrm{RNAP}][\mathrm{Pro}] - k_{\mathrm{off}}[\mathrm{C}]$

   (b)

$$\begin{aligned} K &= \frac{k_{\mathrm{on}}}{k_{\mathrm{off}}} \\ &= \frac{8.5 \times 10^7\,\mathrm{L\,mol^{-1}s^{-1}}}{1.1 \times 10^{-3}\,\mathrm{s^{-1}}} \\ &= 7.7 \times 10^{10}\,\mathrm{L\,mol^{-1}}. \end{aligned}$$

   (c)

$$\begin{aligned} K &= \frac{[\mathrm{C}]}{[\mathrm{RNAP}][\mathrm{Pro}]} \\ \therefore \frac{[\mathrm{C}]}{[\mathrm{Pro}]} &= K[\mathrm{RNAP}] \\ &= (7.7 \times 10^{10}\,\mathrm{L\,mol^{-1}})(1 \times 10^{-6}\,\mathrm{mol\,L^{-1}}) \\ &= 7.7 \times 10^4. \end{aligned}$$

   Based on this calculation, promoters would be overwhelmingly bound with RNA polymerase.

2. If the data fit a 4th order rate law, then a plot of $[\mathrm{Ce}^{4+}]^{-3}$ vs t should be linear. Figure 1 shows the 4th order plot. The line seems to fit the data well, so we could conclude that the reaction obeys 4th order kinetics. I have some reservations about this conclusion, for two reasons:

   (a) We usually like to follow a reaction for at least a couple of half-lives. Otherwise, it may be hard to discriminate between possible orders of reaction. Here, the data cover less than one half-life.

   (b) In this case, we run into exactly that problem: If we generate a 3rd order graph ($[\mathrm{Ce}^{4+}]^{-2}$ vs t), the fit is almost as good.

   If we leave these concerns aside, we can extract a rate constant from the slope of the graph, since

$$\text{slope} = 3k.$$

   The slope of the graph, obtained by linear regression, is

$$\text{slope} = 7.64 \times 10^8\,\mathrm{L^3\,mol^{-3}h^{-1}}.$$

   Therefore,

$$k = \frac{1}{3}(\text{slope}) = 2.55 \times 10^8\,\mathrm{L^3\,mol^{-3}h^{-1}}.$$

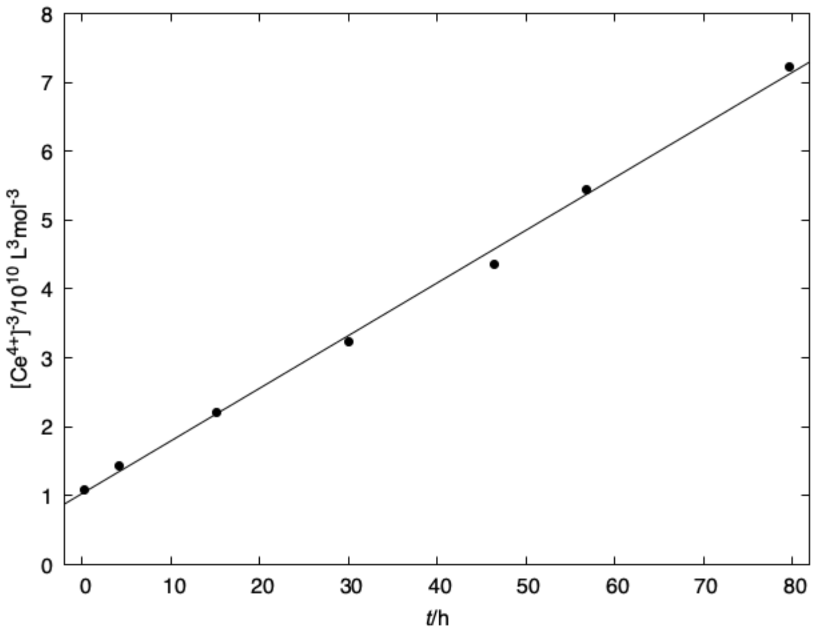

Figure 1: Plot of $[Ce^{4+}]^{-3}$ vs t

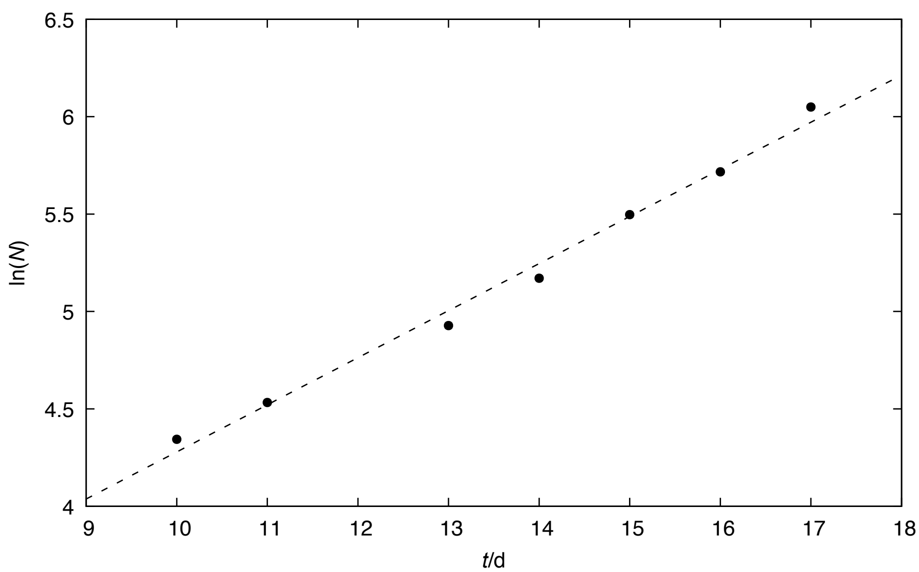

Figure 2: First-order growth plot of the COVID-19 data in Canada

3. (a) To test this, we plot $\ln N$ vs t, where N is the number of cases. My graph is shown in Fig. 2. There are no obvious systematic deviations from the line, so we would conclude that the number of cases really is growing exponentially.

   (b) The slope of the graph is $k = 0.242\,\mathrm{d}^{-1}$. The doubling time is therefore

$$t_2 = \frac{\ln 2}{k} = \frac{\ln 2}{0.242\,\mathrm{d}^{-1}} = 2.87\,\mathrm{d}.$$

   (c) We are looking for the time t at which $N = 0.3(37\,800\,000) = 11\,340\,000$. The equation of the line of best fit is

$$\begin{aligned} \ln N &= (0.242\,\mathrm{d}^{-1})t + 1.86. \\ \therefore t_{30\%} &= \frac{\ln(11\,340\,000) - 1.86}{(0.242\,\mathrm{d}^{-1})} \\ &= 59\,\mathrm{d}. \end{aligned}$$

   The time origin of my graph (day 1) is March 1. The 59th day starting on March 1 is April 28.