Chemistry 2710 Problem Set on Nonlinear Dynamics in One Dimension

The exponential growth model for populations we discussed earlier in class is of course only applicable if there are no constraints to growth. Usually, there will be some limits imposed by the environment. One very popular model used to describe growth with limits is the logistic equation:

$$\frac{dx}{dt} = kx\left(1 - \frac{x}{K}\right).$$

1. Population models often have a corresponding interpretation as chemical models. Show that the mechanism

$$\mathrm{X} + \mathrm{A} \xrightarrow{k_1} 2\mathrm{X},$$

$$2\mathrm{X} \xrightarrow{k_2} \mathrm{P}$$

   is equivalent to the logistic model if A is in great excess such that its concentration can be treated as constant.

2. Find all the equilibrium points of this model.

3. Determine, using methods from nonlinear dynamics, the stability of the equilibrium points.