Chemistry 2710 Solutions to the Problem Set on Nonlinear Dynamics in One Dimension

1. If a is constant, then by the law of mass action,

$$\begin{aligned}\frac{dx}{dt} &= k_1ax - 2k_2x^2 \\ &= k_1ax\left(1 - \frac{2k_2}{k_1a}x\right).\end{aligned}$$

   This is identical in form to the logistic equation if $k = k_1a$ and $K = \dfrac{k_1a}{2k_2}$.

2. In equilibrium,

$$\frac{dx}{dt} = 0 = kx\left(1 - \frac{x}{K}\right).$$

   We get two solutions: $x = 0$ or $1 - x/K = 0$. The latter rearranges to $x = K$.

3. If $x < 0$, $\dot{x} < 0$. If $0 < x < K$, $\dot{x} > 0$. Finally, if $x > K$, $\dot{x} < 0$. The one-dimensional vector field therefore has the following appearance:

   The equilibrium point at $x = 0$, represented by an open circle, is unstable. On the other hand, the equilibrium point at $x = K$, the filled circle, is stable. Thus we expect that for any positive initial population, the population will stabilize at the level $x = K$. Because of this, the constant K is usually called the carrying capacity in works on population dynamics.