Chemistry 2710 Spring 2004 Test 1 Solutions

1. (a) If 95% has decayed, then 5% remains:

$$\begin{aligned}
\frac{x}{x_0} &= \left(\frac{1}{2}\right)^{t/t_{1/2}}. \\
\therefore \ln\left(\frac{x}{x_0}\right) &= \frac{t}{t_{1/2}}\ln\left(\frac{1}{2}\right). \\
\therefore t &= (5.3\,\text{s})\frac{\ln 0.05}{\ln\left(\frac{1}{2}\right)} = 23\,\text{s}.
\end{aligned}$$

(b)

$$\begin{aligned}
n_{\text{Tc}} &= \frac{0.0432\,\text{g}}{101.909\,213\,\text{g/mol}} = 4.24\times10^{-4}\,\text{mol}. \\
k &= \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5.3\,\text{s}} = 0.13\,\text{s}^{-1}. \\
\therefore -\frac{dn_{\text{Tc}}}{dt} = \frac{dn_{\beta}}{dt} &= kn_{\text{Tc}} = (0.13\,\text{s}^{-1})(4.24\times10^{-4}\,\text{mol}) = 5.54\times10^{-5}\,\text{mol/s}.
\end{aligned}$$

2. The best way to determine whether the reaction follows a simple rate law is to use a van't Hoff plot:

$\ln[NO_3^-]$	3.9060	4.3883	4.6151	5.0173
$\ln v$	0.8242	1.4422	1.7750	2.2659

Figure 1 shows the plot: The data fit a straight line, so the reaction follows a simple rate law. The order of the reaction is the slope of the van't Hoff plot. By linear regression, we find an order of 1.30.

3. (a) In a dissolution process, the mass decreases, so the rate would be negative.

(b)

$$\begin{aligned}
\frac{dm}{m^{2/3}} &= -k\,dt. \\
\therefore \int_{m_0}^{m} m^{-2/3}dm &= -k\int_0^t dt. \\
\therefore 3m^{1/3}\Big|_{m_0}^{m} &= -kt. \\
\therefore 3m^{1/3} - 3m_0^{1/3} &= -kt. \qquad (1)
\end{aligned}$$

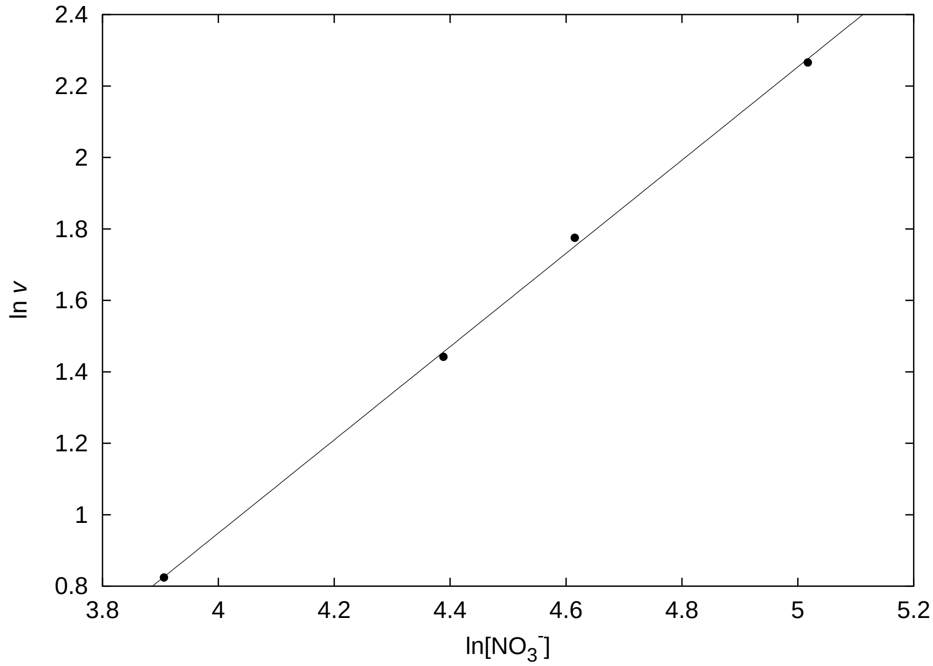

Figure 1: van't Hoff plot for the data of question 2.

(c) Rearrange equation 1 to the form

$$m^{1/3} = m_0^{1/3} - \frac{1}{3}kt.$$

If we plot $m^{1/3}$ vs t, we should get a straight line of slope $-\frac{1}{3}k$. The rate constant is therefore $k = -3 \times (\text{slope})$.

(d) k would have units of $\text{g}^{1/3}/\text{s}$.

4. If bp decays with first-order kinetics, a plot of $\ln[\text{bp}]$ vs t should be linear. The % bp (hereafter denoted p) is proportional to the concentration and can therefore be used in our plot.

t (min)	2.21	5.16	13.26	22.84	40.52	60.41
$\ln p$	4.5539	4.4188	4.1744	3.7842	3.1355	2.3026

The first-order plot is shown in Figure 2. There is no obvious deviation from linearity, so the data are consistent with a first-order reaction. The slope of the graph is $-0.0383\,\text{min}^{-1}$ (obtained by linear regression), so the rate constant is $k = 0.0383\,\text{min}^{-1}$. The half-life is therefore

$$t_{1/2} = \frac{\ln 2}{k} = \frac{\ln 2}{0.0383\,\text{min}^{-1}} = 18\,\text{min}.$$

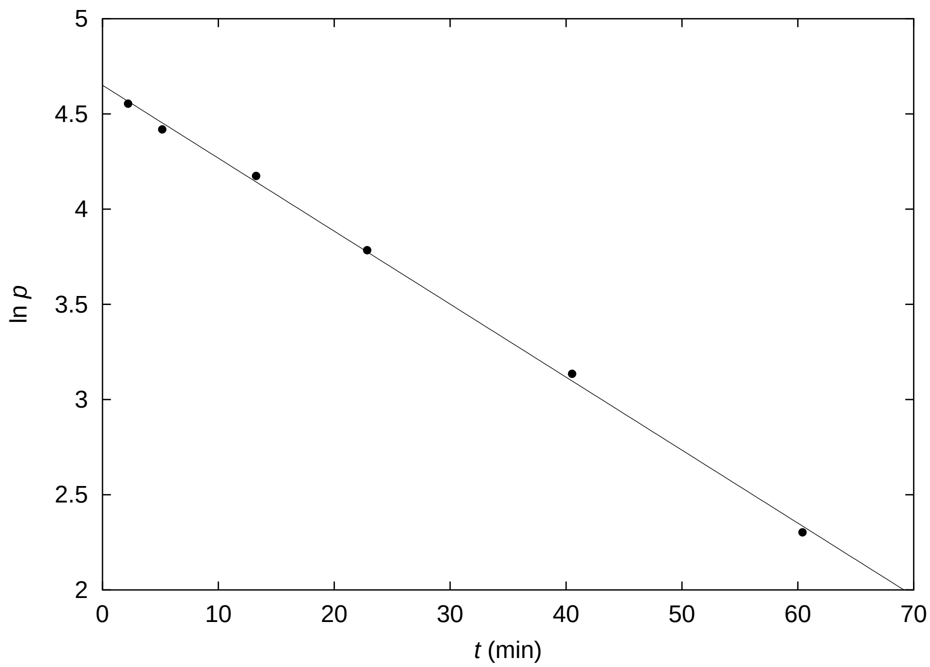

Figure 2: First-order plot for the data of question 4.