## Economics 3950 <br> Spring 2005

## Dr. Richard Mueller

## Assignment \#4

Instructions: These questions should be answered using a text editor or a word processor where you can cut and paste output from your statistical program (where necessary). Please mark question numbers clearly. This assignment is due on Monday, March 30, 2008 by 12:00 in D-552.

1. (70 points total) Exercise 7.7, pp. 330-1.
2. (30 points total) Exercise 7.22, pp. 337-8.
3. (35 points total) Exercise 8.21, pp. 374-5.

## Grand Total: 135 points

## Answer Key

1. a. The control season is Winter because no dummy variable has been defined for it.
b. When the price of a car goes up, we would expect sales to decline so that $\beta<0$, Income will have a positive effect so that $\alpha>0$. If the interest rate increases, the cost of financing a car will go up and hence the demand for cars will decrease. Thus, we would expect $\delta<0$.
c. Let $\alpha=\mathrm{a} 0+\mathrm{a}$ SPRING +a 2SUMMER +a 3 FALL. Substituting this in the basic model we get the unrestricted Model ( U) as LPCCARS $=\mathrm{a} 0+\mathrm{a}$ 1SPRING +a 2SUMMER +a 3 FALL $+\beta$ LPRICE + $\gamma$ LPINCOME $+\delta$ LPINTRATE + error. This is the same as Model B.
d. $\mathrm{Ho}: \mathrm{a} 1=\mathrm{a} 2=\mathrm{a} 3=0 . \mathrm{H} 1$ : At least one term is nonzero.
e. Compute $\mathrm{Fc}=[(0.31044-0.17699) / 3] /[0.17699 /(40-7)]=8.29$.
f. $\mathrm{Fc} \sim \mathrm{F}(3,33)$
g. From the F-table for 1 per cent level $\mathrm{F}^{*}$ with 3 and 33 d.f. is a value between 4.31 and 4.51.
h. Since $\mathrm{Fc}>\mathrm{F}^{*}$ we can reject the null hypothesis.
i. To decide on the variables to exclude, we can do a t-test. The degrees of freedom for the t -statistics in Model B in the table is 33 . For a $1(10)$ per cent level of significance, $\mathrm{t}^{*}$ is between 2.704 and 2.750 ( 1.684 and 1.697). Any variable with a corresponding $t$-statistic below this is a candidate to be dropped. According to this rule we would drop SUMMER and FALL and this would result in Model C. Of course, if we look at $t$-statistics with absolute values of less than 1 , we couldn't drop anything.
j. The model with the lowest model selection statistics is preferable as the best. Six out of the eight criteria choose Model C as the best. Another criterion to use is the significance of the coefficients. All the coefficients in Model C are highly significant. Thus, Model C would be chosen as best model.
k. Let $\beta=\mathrm{b} 0+\mathrm{b} 1$ SPRING +b 2 SUMMER +b 3 FALL. Substituting this in Model B we get the unrestricted Model D as LPCCARS $=\mathrm{a} 0+\mathrm{a}$ 1SPRING +a 2 SUMMER +a 3 FALL +boLPRICE +b1SPRING*LPRICE +b 2 SUMMER*LPRICE +b 3 FALL*LPRICE $+\gamma$ LPINCOME + $\delta$ LPINTRATE + error.
2. $\mathrm{Ho}: \mathrm{b} 1=\mathrm{b} 2=\mathrm{b} 3=0$.
m. First regress LPCCARS against all the explanatory variables in Model B and save the error sum of squares as ESSB. Next create Z1 $=$ SPRING*LPRICE, Z2 $=$ SUMMER*LPRICE, and Z3 $=$ FALL*LPRICE. The regress LPCCARS against the variables in model B plus the three Zs and save the error sum of squares as ESSD. Next compute $\mathrm{Fc}=[(\mathrm{ESSB}-\mathrm{ESSD}) / 3] /[\mathrm{ESSD} /(40-10)]$. Under the null $\mathrm{Fc} \sim \mathrm{F}(3,30)$. The the F -table for 1 per cent look up $\mathrm{F}^{*}=4.51$. If $\mathrm{Fc}>4 . .51$ we would reject the null hypotheses.
n. Ignoring the negative sign of the coefficient, computer $\mathrm{tc}=(\beta$ hat -1$) /$ (std. error). Since the standard error is not directly given, we obtain is as std. error $=$ coeff/t-stat $=1.76 / 9.6=0.183$. From this we have tc $=(1.760-1) / 0.183=4.15$. Under the null hypothesis this has $40-5=35$ d.f. and we can reject the hull hypothesis. Thus, since the observed price elasticity is greater than 1 , we can say that demand is price elastic is these data.
3. a. Ho: $\beta_{6}=\beta_{7}=\beta_{8}=\beta_{9}=\beta_{10}=0$.
b. Model A is the unrestricted model and Model B is the restricted model. Compute: $\mathrm{Fc}=[$ (ESSBESSA $) / 5] /[(\mathrm{ESSA}) /(116-10)]=122.51$. Under $\mathrm{Ho}, \mathrm{Fc} \sim \mathrm{F}$ with 5 and 106 d.f.
c. From the F table for 1 per cent level, F is between 3.17 and 3.34 . Since $\mathrm{Fc}>\mathrm{F}^{*}$ we reject Ho and conclude that there has been a significant change in the structure.
d. Six of out the eight model section criteria choose Model C as the best. But Model C has $\beta_{2}$ and $\beta_{8}$ with $p$-values slightly above 10 per cent. Omitted variable bias suggests that it is better to leave a variable in a model if it appears to have some effect. Since these two are only marginally insignificant, Model C is best.
e.

|  | 1980 | 1990 |
| :--- | :--- | :--- |
| FAMSIZE | 4.944 | $4.944+9.760=14.704$ |
| HIGHSCHL | 0.223 | $0.223+0.199=0.422$ |
| COLLEGE | 0.339 | $0.339+0.871=1.210$ |

f. In 1990 and increase in family size of one person resulted in average increase of $\$ 14,704$ in median income where this was only $\$ 4,944$ in 1980.

A one per cent increase in high school graduates increased median income on average by $\$ 422$ in 1990, which is $\$ 199$ more than in 1980.

A one per cent increase in college graduates increase median income on average of \$339 in 1980 and $\$ 1,210$ in 1990.
3. a.
$\sigma_{t}^{2}=\alpha_{o}+\alpha_{1} E D U C_{t}+\alpha_{2} U E_{t}+\alpha_{3} D R_{t}+\alpha_{4} U R B_{t}+\alpha_{5} W H_{t}+\alpha_{6} U E_{t}^{2}+\alpha_{7} D R_{t}^{2}+\alpha_{8} U R B_{t}^{2}+\alpha_{9} W H_{t}^{2}$
b. Ho: $\alpha_{i}=0$ for $i=1,2, \ldots, 9$
c. $\quad \mathrm{LM}=\mathrm{nR}^{2}=50 *[1-(888.498 / 1725.33)]=24.25$
d. Under the null hypothesis, $L M \sim X_{9}^{2}$.
e. $L M^{*}(0.01)=21.666$. Since $\mathrm{LM}>\mathrm{LM}^{*}$, we reject Ho and conclude that there is significant heteroscedasticity.
f. Ignoring heteroscedasticity, as OLS does, does not jeopardize unbiasedness nor consistency, but OLS estimates are no longer efficient and hypothesis tests are invalid.
g. i. Compute

$$
\begin{aligned}
\hat{\sigma}_{t}^{2}= & 105.356-.273 E D U C_{t}-6.183 U E_{t}-5.332 D R_{t}-.883 U R B_{t}-1.001 W H_{t}+.557 U E_{t}^{2}+.307 D R_{t}^{2} \\
& +.007 U R B_{t}^{2}+.011 W H_{t}^{2}
\end{aligned}
$$

ii. Compute $w_{t}=1 / \sqrt{\hat{\sigma}_{t}^{2}}$
iii. Regress $w_{t} W L F P_{t}$ against all non-squared terms weighted by $w_{t}$ in the equation in part (a) and leave out the constant.

