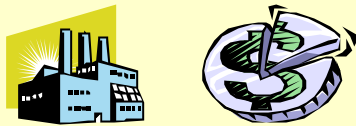


Economics 3030

Chapter 5

The Production Process and Costs



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Overview

I. Production Analysis

- Total Product, Marginal Product, Average Product
- Isoquants
- Isocosts
- Cost Minimization

II. Cost Analysis

- Total Cost, Variable Cost, Fixed Costs
- Cost Relations

III. Multi-Product Cost Functions

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Production Analysis

- Production Function
 - $Q = F(K, L)$
 - The maximum amount of output that can be produced with K units of capital and L units of labour.
 - K & L are factors of production.
- Short-run (fixed capital) vs. long-run (all factors variable) decisions.

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Total Product

- Example: Cobb-Douglas Production Function
- $Q = F(K, L) = K^{.5} L^{.5}$
 - K is fixed at 16 units.
 - Short run production function:
 $Q = (16)^{.5} L^{.5} = 4 L^{.5}$
 - Production when 100 units of labour are used?
 $Q = 4 (100)^{.5} = 4(10) = 40$ units
- TP is the maximum amount that can be produced with a given quantity of inputs.

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Marginal Product of Labour

- $MP_L = \Delta Q / \Delta L$
- Measures the output produced by the last worker.
- Slope of the production function
- MP_L increases as L is added, reaches a maximum, and then decreases (Diminishing Marginal Returns).

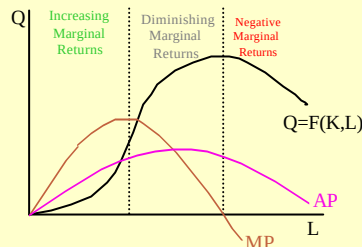
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Average Product of Labor

- $AP_L = Q/L$
- Measures the output of an “average” worker, or output per worker.

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Stages of Production



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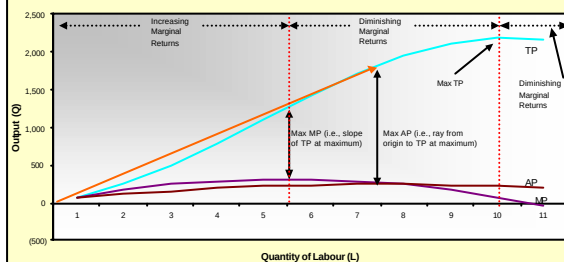
A Numerical Example

Table 5-1: The Production Function

(1) Fixed Input (Capital) [Given]	(2) Variable Input (Labour) [Given]	(3) Change in Labour [Δ (2)]	(4) Output [Given]	(5) Marginal Product of Labour [Δ (4)/Δ (2)]	(6) Average Product of Labour [(4)/(2)]
2	0	--	0	--	--
2	1	1	76	76	76
2	2	1	248	172	124
2	3	1	492	244	164
2	4	1	784	292	196
2	5	1	1,100	316	220
2	6	1	1,416	316	236
2	7	1	1,708	292	244
2	8	1	1,952	244	244
2	9	1	2,124	172	236
2	10	1	2,200	76	220
2	11	1	2,156	-44	196

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The Production Function (from Table 5.1)



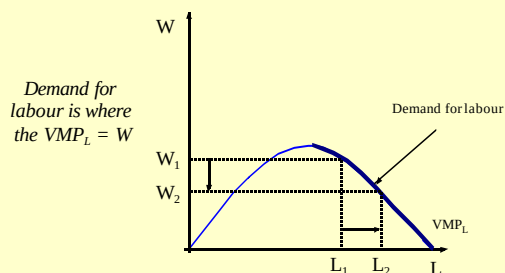
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How much labour should be hired?

- It is important that the “correct” amount of inputs are hired.
- In the short term, capital is fixed. Therefore, only the amount of labour that should be hired enters the decision process.
- Value of the Marginal Product of Labour
 - $VMP_L = MP_L \cdot P$ of output

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Demand for Labour Curve



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Long-Term Production

- In the short term, L is the only variable factor of production.
- In the long term, both K and L are variable.
- As a result, decisions about the appropriate amounts of both labour and capital must be made.

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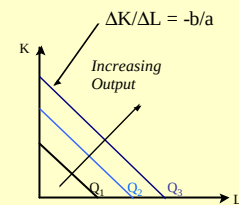
Isoquant

- The combinations of inputs (K, L) that yield the producer the same level of output.
- The shape of an isoquant reflects the ease with which a producer can substitute among inputs while maintaining the same level of output.

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Linear Isoquants

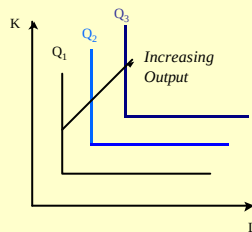
- $Q = F(K, L) = aK + bL$
- Capital and labour are perfect substitutes
- A unit of capital is as good as a unit of labour (and vice versa).



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Leontief Isoquants

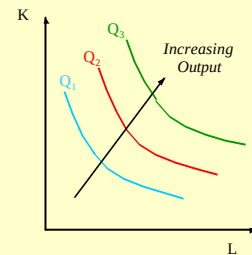
- $Q = F(K, L) = \min(bK, cL)$
- Capital and labour are perfect complements
- Capital and labour are used in fixed-proportions
- Additional L or K does not add to total output



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Cobb-Douglas Isoquants

- $Q = F(K, L) = K^a L^b$
- Inputs are not perfectly substitutable
- $MRTS_{KL} = MP_L / MP_K$
- Diminishing marginal rate of technical substitution
- Most production processes have isoquants of this shape



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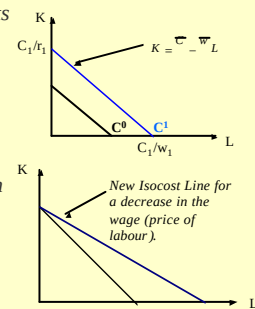
Note: The MRTS is

- 0 for Leontief isoquants (no substitutes)
- constant for linear isoquants (perfect substitutes)
- diminishing for Cobb-Douglas isoquants (less-than-perfect substitutes)
- The latter case implies that K and L are substitutable, but increasing amounts of one are needed as we use less of the other

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Isocost

- The combinations of inputs that cost the producer the same amount of money
- $C = wL + rK$
- For given input prices, isocosts farther from the origin are associated with higher costs.
- Changes in input prices change the slope of the isocost line



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Cost Minimization

- Marginal product per dollar spent should be equal for all inputs:

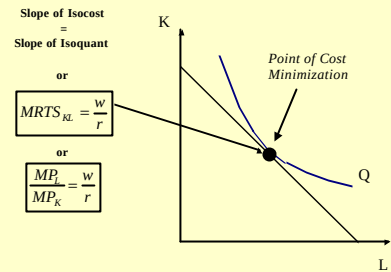
$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- Expressed differently

$$MRTS_{KL} = \frac{w}{r}$$

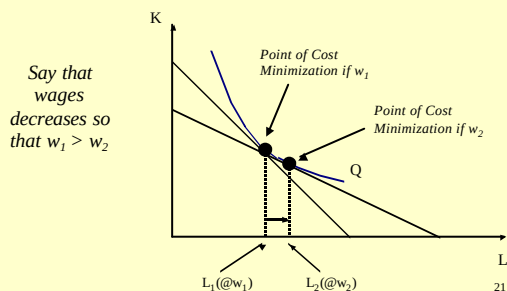
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Cost Minimization (2)



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Changing Factor Prices



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Cost Analysis

- Types of Costs
 - Fixed costs (FC)
 - Variable costs (VC)
 - Total costs (TC)
 - Sunk costs



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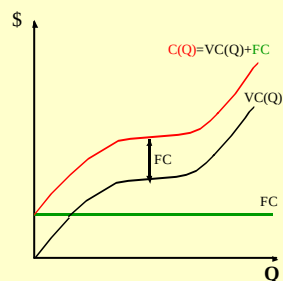
Short-Term Total and Variable Costs

$C(Q)$: Minimum total cost of producing alternative levels of output:

$$C(Q) = VC(Q) + FC$$

$VC(Q)$: Costs that vary with output

FC : Costs that do not vary with output



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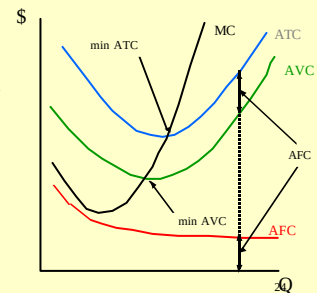
Short-Term Average & Marginal Costs

- Average Total Cost
 - $ATC = AVC + AFC$
 - $ATC = C(Q)/Q$

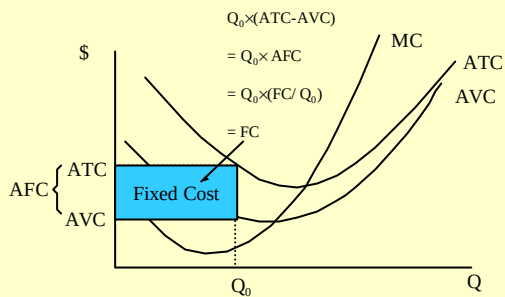
- Average Variable Cost
 - $AVC = VC(Q)/Q$

- Average Fixed Cost
 - $AFC = FC/Q$

- Marginal Cost
 - $MC = \Delta C / \Delta Q$

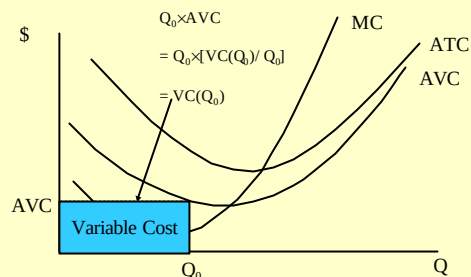


Short-Term Fixed Cost



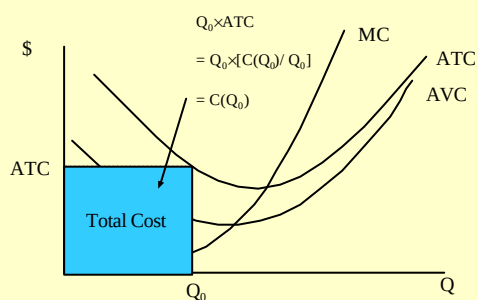
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Short-Term Variable Cost



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Short-Term Total Cost



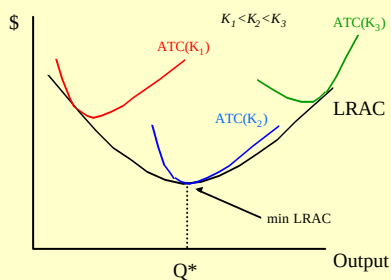
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Long-Term Costs

- In the long-term, K and L are both variable.
- Thus, all costs are variable in the long-term.
- We now have a long-run average cost (LRAC) curve.
- The LRAC curve “envelopes” a series of short-run ATC curves. Each of these represents a different level of capital.

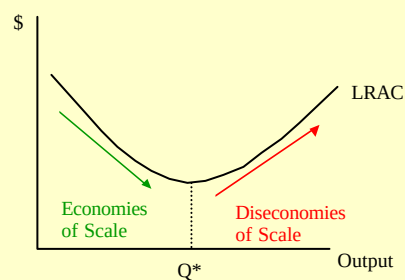
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Long-Term Average Cost



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Economies of Scale



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Multi-Product Cost Function

- $C(Q_1, Q_2)$: Cost of producing two outputs jointly

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Economies of Scope

- $C(Q_1, Q_2) < C(Q_1, 0) + C(0, Q_2)$
- It is cheaper to produce the two outputs jointly instead of separately.
- Example: Toyota Sienna and Camry

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Cost Complementarity

- The marginal cost of producing good 1 declines as more of good two is produced:

$$\Delta MC_1 / \Delta Q_2 < 0.$$

- Examples:
 - Donuts and donut holes
 - Oil and natural gas production
 - Today's cafeteria entrée and tomorrow's soup

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Summary

- Various combinations of capital and labour can be used to produce a given level of output.
- Costs associated with these differ in the long term and the short term (when only labour is a variable factor).
- Economies of scale, economies of scope, and cost complementarities in production often exist.

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