Projecting Cartesian Coordinates onto the Surface of an Egg in Egg-bot Coordinates

In order to plot an image on the surface of an egg in such a way that it does not look distorted when viewed straight-on, the original Cartesian image coordinates must be projected onto the curved surface, and the projected curve coordinates must be converted to the spherical polar coordinate system of the Egg-bot. The figure below shows a rectangular image to be projected onto an egg-shaped surface.

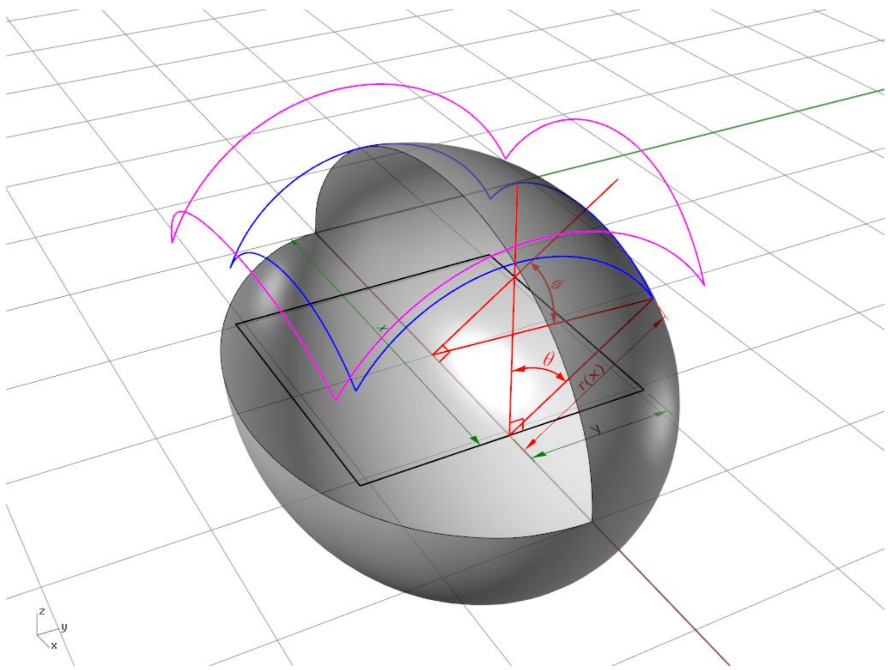

Figure 1. Cartesian and Egg-bot coordinate systems for an egg-shaped surface (1/4 cut away for clarity). The black square is projected along the z axis to produce the blue curve on the egg surface. When the egg is viewed from the top, the blue curve appears to be a square. The magenta curve represents the path in polar coordinates on a sphere required to plot the blue curve on the egg. The polar coordinates are shown in red for one corner of the projected square.

To determine the polar coordinates in the Egg-bot coordinate system for the projection of cartesian coordinate [*x,y*] onto the egg surface (in the z direction), the egg radius $r_{(x)}$ must be known as a function of the distance along the long axis. Equation 1 below is a beautiful equation (derived by Nobuo Yamamoto) giving the radius of any egg-like surface over the full range of possible eccentricities, as a function of the distance along the long axis, *x*:

$$r_{(x)} = \sqrt{\frac{x}{2}}\sqrt{(a-b) - 2x + \sqrt{4bx + (a-b)^2}} \qquad \text{Eq. 1}$$

The parameter *a* is the length of the long axis, and *b* determines the eccentricity of the surface. In the figure above, *a* = 4 and *b* = 0.7**a*. The parameter *b* must not be greater than *a*. The pointy end of the egg is towards *x* = 0. The midpoint is *x* = *a*/2, which is also the center of rotation for both Egg-bot axes.

Now, if the pen is located above the egg on the *xz* plane, with its axis of rotation at *x* = *a*/2, then in order to plot the projection of point [*x,y*] on the surface, the egg rotation angle is given by:

$$\theta = Sin^{-1}\left(\frac{y}{r_{(x)}}\right) \qquad \text{Eq. 2}$$

and the pen rotation angle is given by:

$$\emptyset = Tan^{-1}\left(\frac{x - a/2}{r_{(x)}}\right) \qquad \text{Eq. 3}$$

These equations are more complex than those for projecting onto a sphere, due to the eccentricity of the egg. Another way of thinking of this is that when viewed from the top, the projected image on the egg is square, but the same polar coordinates plotted on a sphere are not, as shown below. Conversely, plotting an image that appears square on a spherical surface onto an egg will result in a curved image.

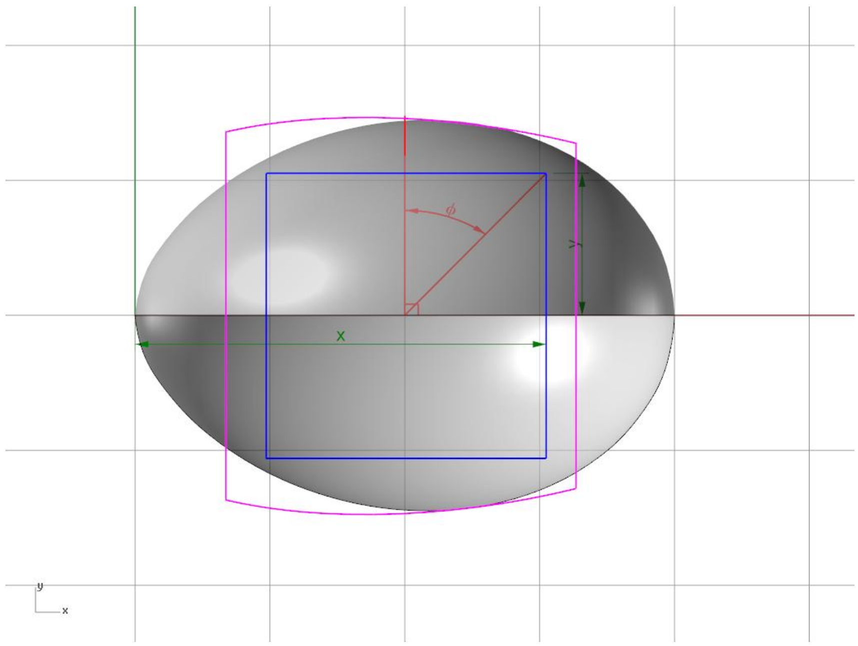

Figure 2. Top view of square projected onto egg surface (blue) and path on sphere with the same polar coordinates (magenta).

Matching Model Parameters to Egg Measurements

The parameter *a* is the length of the egg, and the parameter *b* is determined by the eccentricity of the egg. Given the ratio of the measured maximum diameter of the egg to the length, Eq. 1 above can be solved for its maximum value to determine the ratio of *b* to *a*. The figure below allows *b* to be determined given *a* and the maximum diameter of the egg. For a typical chicken egg, the ratio of *b* to *a* is 0.7. For a sphere, b is zero.

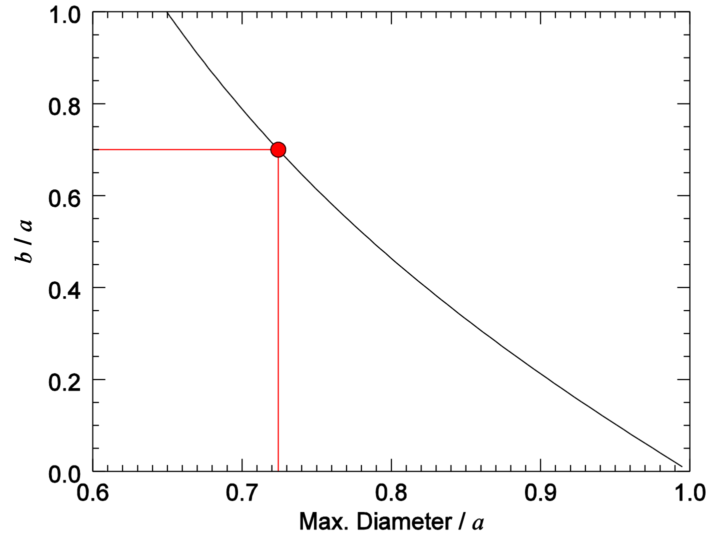

Figure 3. Look-up table for *b / a* ratio given measured egg dimensions.

Plotting Line Segments of Equal Length on an Egg

The Egg-bot coordinate system [θ, $\emptyset$] is spherical, but the egg surface does not have constant radius. This means that the length of a 1-degree line segment varies depending on what part of the egg it is drawn on. Equations 2 and 3 can be differentiated to determine the projected arc length for a given rotation of the egg or pen axis.

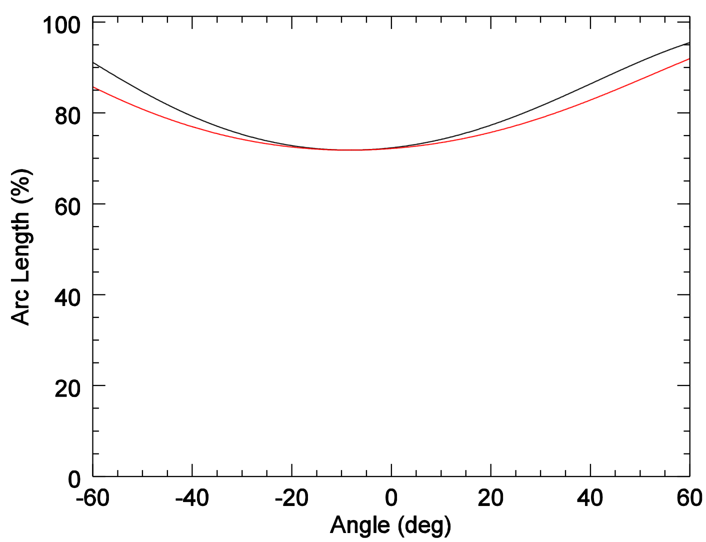

Figure 4. Longitudinal arc length per step of pen rotation (black), and latitudinal arc length per step of egg rotation, as a function of pen rotation angle, relative to that for a sphere.

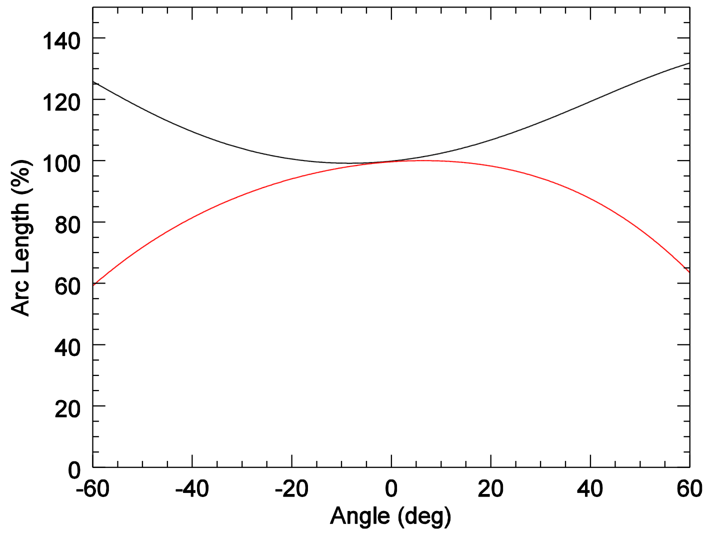

Figure 5. Arc length per step relative to that at the equator of the egg, for the pen coordinate (black) and egg rotation coordinate (red), as a function of pen rotation angle.

For example, when the pen is at the pointy end of the egg (-60°), rotating the pen by one step produces a line segment that is 125% what it would be at the equator of the egg. Rotating the egg by one step, however, produces a line only 60% as long as it would at the equator. This is a result of the facts that the diameter of the egg obviously gets smaller at the ends, but also, the surface of the egg deviates from the circular arc of the pen more near the equator of the egg. So, in order to plot text around the top end of an egg so that it appeared to have the same font size as text plotted around the equator, the text would need to be scaled by 0.8 vertically and 1.67 horizontally. This assumes of course that the text size is small enough that a pair of scaling factors is sufficient.

Another way of thinking of this distortion is by comparing the arc length per step on an egg surface to that on a sphere with a diameter of *a*, as shown below.

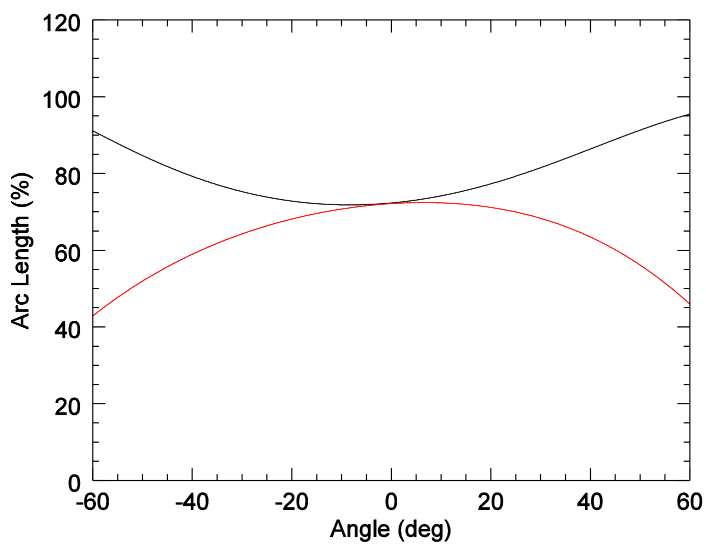

Figure 6. Arc length per step relative to that at the equator of a sphere, for the pen coordinate (black) and egg rotation coordinate (red), as a function of pen rotation angle.

For example, when the pen is at the pointy end of the egg (-60°), rotating the pen by one step produces a line segment that is nearly the same as what it would be on a sphere with diameter equal to the length of the egg. Rotating the egg by one step, however, would produce a line only ~40% as long as it would at the equator of that sphere. Also, rotating the egg by one step when the pen is at the equator results in a line only ~75% as long as on a sphere, since the equator of the sphere is larger than that of the egg.

Plotting Straight Lines on an Egg

Plotting a straight latitude or longitude line around an egg is trivial. Plotting a line parallel to a longitude line on an egg, as shown below, is more difficult.

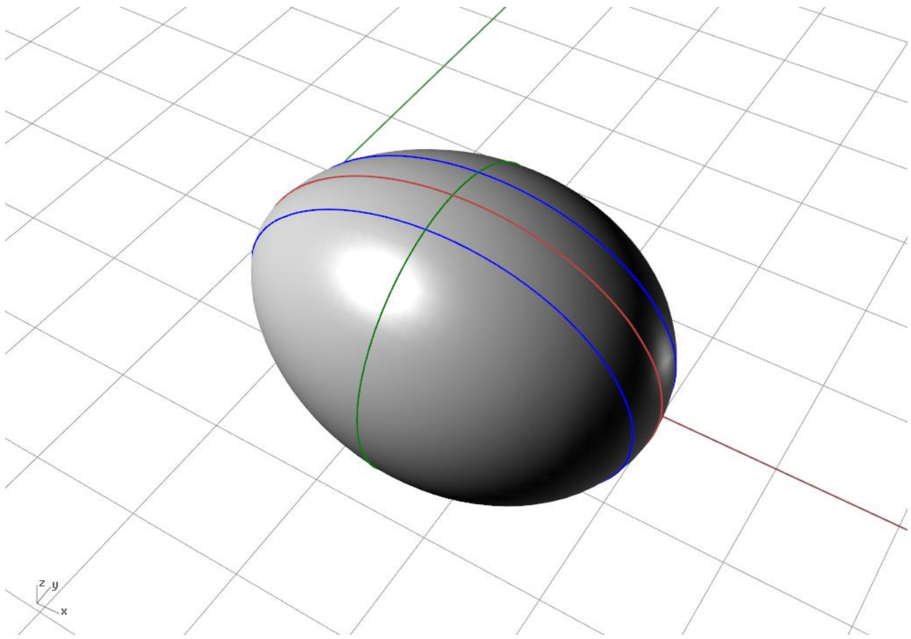

Figure 7. Equator and meridian lines (green and red) are easily plotted using one axis of the Egg-bot. Plotting lines parallel to a longitudinal line (blue) requires rotation around both axes.

The process of computing a parallel line to a meridian line is equivalent to projecting a line, which lies on the Cartesian plane parallel to the x axis and with ends that intersect with the egg surface, onto the surface of the egg. From Equation 1 above, the Cartesian x coordinates for the endpoints of a line a distance *y* from the x axis would therefore be the solutions to $r_{(x)} = y$. The Egg-bot coordinates would be computed by evaluating Equations 2 and 3 over this range of *x*, holding *y* constant. In order to plot the line all the way around the egg, the path would need to be repeated, with $2(90° - \theta)$ added to the θ coordinates.

For example, the θ and $\emptyset$ coordinates for drawing a line at *y* = 1 are shown below. The red curve is the plot of Equation 3 vs. Equation 2. The blue curve is the reverse of the first curve, with $2(90° - \theta)$ added to the θ coordinates. Joining these paths together will produce a continuous line around the egg, parallel to the meridian at $\theta = 0$.

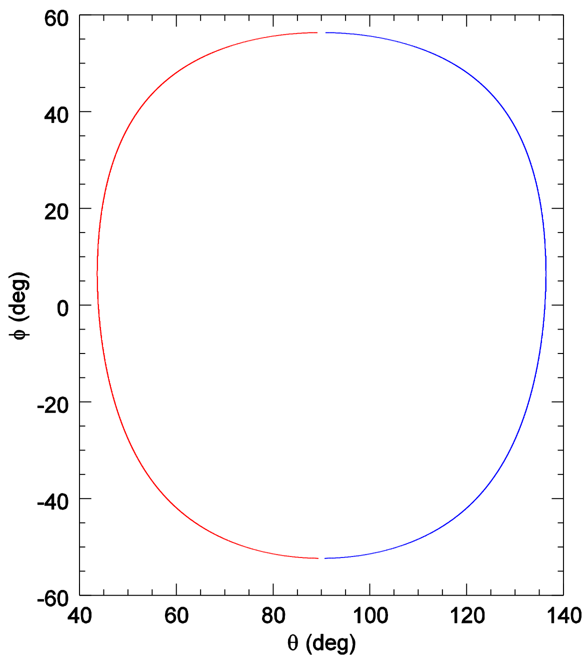

Figure 8. Polar coordinates required to plot a line parallel to the meridian with y = 1.