Chemistry 5850 Fall 2005 Assignment 10

Due: Monday, Nov. 28.

Weight of this assignment: 49 marks

1. Unstable fixed points of maps typically can be reached in a finite number of steps from special initial conditions. To find these special initial conditions, we compute the pre-image of a fixed point, i.e. the set of points which lead in a single iteration to the fixed point. Then we compute the pre-images of those points, and so on until we run out of patience or computer resources.

    (a) Calculate the Liapunov exponent of the logistic map for $\lambda = 4$. Comment on the significance of the value obtained. [2 marks]

    (b) Calculate all nth pre-images of the non-extinction fixed point for n up to 3. Generalize your result if possible. [5 marks]

    (c) What happens if you take an initial condition very close (but not identical) to one of the 3rd pre-images? [2 marks]

    (d) Find an unstable periodic orbit and compute its pre-images. [5 marks]

2. We can think of the logistic map as a special case of the map

$$x_{n+1} = \lambda x_n^a \left(1 - x_n\right)^a .$$

    (a) Is the logistic map structurally stable? In other words, if we take a close to 1, do we get similar behavior? [5 marks]

    (b) What happens for values of a significantly different from 1? Consider integer, rational and irrational values of a in exploring this system. [30 marks]