Chemistry 5850 Fall 2005 Assignment 1

Due: Monday, September 12.

Weight of this assignment: 48 marks

1. Is the stock market a dynamical system? Discuss.

   In a question like this one, you are expected to consider different viewpoints, but you should eventually take a stand on on one side or the other of the issue.

   This is, incidentally, not an entirely academic question. Some methods for treating data from dynamical systems do not require that we know the underlying evolution equations. However, these methods are only valid if the system really is a dynamical system, i.e. if it satisfies the definition discussed in class. There have been a number of attempts to use dynamical systems theory to predict the behavior of the stock market. It's difficult to tell how successful these attempts have been because the more sophisticated ones have typically been carried out by companies selling their market analysis services, and have been shrouded in secrecy. [10 marks]

2. In this problem, we will build a simple model of bacterial growth in a *chemostat*, a simple device for maintaining (or attempting to maintain) constant growth conditions. You will then apply phase-plane methods to this model.

   The chemostat is illustrated in Fig. 1. A nutrient solution is pumped in through the inlet port. The nutrients are consumed by the bacteria within the chemostat, supporting bacterial growth and reproduction. The chemostat is vigorously stirred (several hundred rpm). If the flow rate at the inlet port is f (in, e.g., L/h), then the flow rate through the outlet port must also be f. A mixture of cells, unconsumed nutrients, and metabolic products exits through the latter port.

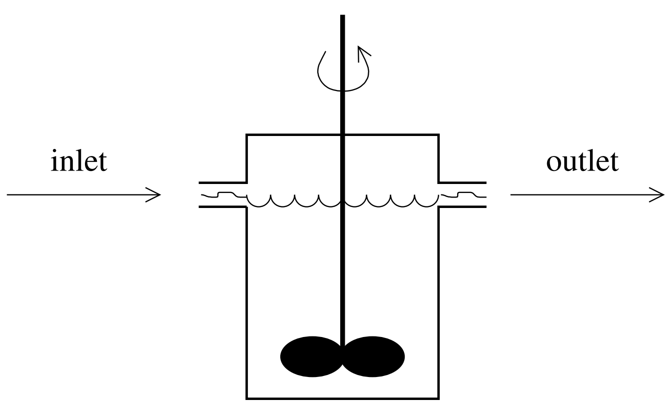

Figure 1: Sketch of a chemostat, otherwise known as a continuous stirred tank reactor (CSTR).

For simplicity, we will treat the case of a single nutrient, although typical nutrient solutions used in chemostats are a real witch's brew. The single-nutrient model is reasonable if one of the nutrients is limiting.

(a) Given the flow rate f, a nutrient concentration c_0 in the feed solution, and a chemostat of working volume V, derive an equation for the rate of change of the nutrient concentration, c, due solely to the nutrient being pumped in. [5 marks]

(b) Derive an equation for the rate of change of the concentration of nutrient due to outflow. Note that a similar equation must apply to the biomass. [2 marks]

(c) Uptake of nutrients by bacteria typically follows the Michaelis-Menten law:

$$\text{uptake rate} = \frac{kbc}{c+K},$$

where k and K are constants, and b is the biomass. Write a rate equation for the rate of change of c due to all three processes (flow in, flow out, and uptake by cells). [2 marks]

(d) The rate of growth of biomass is typically found to be proportional to the rate of nutrient uptake. The proportionality constant is called the biomass yield, and is usually denoted Y. Write a rate equation for the biomass, taking into consideration biomass growth, outflow, and senescence (cell death). Assume (for simplicity) that the rate of senescence is directly proportional to the biomass concentration. [4 marks]

(e) You now have two coupled rate equations. Transform them to dimensionless form. [5 marks]

(f) Carry out a complete phase-plane analysis of this system. [20 marks]