Statistical Mechanics Assignment 7

Due: April 3, 11:00 a.m.

Marks: 13

1. A typical value for the diffusion coefficient of a small molecule in solution is $2 \times 10^{-9}\,\mathrm{m}^2/\mathrm{s}$. If we apply the gas-phase theory to a molecule of molar mass $50\,\mathrm{g/mol}$ with a diffusion coefficient of this size at 25°C, what value do we calculate for the mean free path? Does this make any physical sense? [4 marks]

2. The diffusion equation has a number of solutions other than the one seen in class. Here is one:
$$c(x,t) = \alpha e^{-\pi^2 Dt/L^2} \sin(\pi x/L).$$
   (a) Show that this solution satisfies the diffusion equation. [2 marks]
   (b) What boundary conditions does this solution satisfy at $x = 0$ and $x = L$? [2 marks]
   Note: One possible geometry associated with a solution like this one consists of a long thin pipe of length L. The chemical that diffuses in the pipe is rapidly removed once it reaches either end of the pipe.
   (c) What is the flux from each end of the pipe? [3 marks]
   (d) If the pipe has cross-sectional area A, what is the total rate of loss of material from the pipe? [2 marks]

Bonus: The form given above can be generalized by taking a sum of terms of this form with different coefficients in the exponential and sine function while keeping the same boundary conditions. Write down an equation for a general term in this series.