Chemistry 2720 Fall 2005 Test 2 Solutions

- 1. The information given suggests that the imine condensation is thermodynamically unfavorable on its own, i.e. $\Delta \bar{G}^{\circ}$ is large and positive. By coupling this reaction with the hydrolysis of trimethylorthoformate, a reaction which is presumably strongly favored, i.e. one which has a large negative $\Delta \bar{G}^{\circ}$, we get a reaction with an overall negative $\Delta \bar{G}^{\circ}$. The coupling in this case is achieved by the removal of water by reaction with the solvent trimethylorthoformate.
- 2. The reaction is

$$\operatorname{CaF}_{2(\mathrm{s})} \to \operatorname{Ca}^{2+} + 2\mathrm{F}_{(\mathrm{aq})}^{-}.$$

$$c = \frac{0.017 \,\mathrm{g/L}}{78.07 \,\mathrm{g/mol}} = 2.2 \times 10^{-4} \,\mathrm{mol/L}.$$

$$\therefore [\operatorname{Ca}^{2+}] = c = 2.2 \times 10^{-4} \,\mathrm{mol/L},$$

$$\operatorname{and} [\mathrm{F}^{-}] = 2c = 4.4 \times 10^{-4} \,\mathrm{mol/L}.$$

$$\therefore K_{\mathrm{sp}} = (a_{\mathrm{Ca}^{2+}})(a_{\mathrm{F}^{-}})^{2} = (2.2 \times 10^{-4})(4.4 \times 10^{-4})^{2} = 4.1 \times 10^{-11}.$$

So far, we haven't made any approximations, except insofar as we are ignoring non-ideal effects. The solubility is given at 26°C. Because this is so close to 25°C, and because there is no way to adjust the data to this temperature with the data we have available, I'm going to treat our $K_{\rm sp}$ as if it had been measured at 25°C.

$$\begin{split} \Delta \bar{G}^{\circ} &= -RT \ln K_{\rm sp} = -(8.314\,472\,\mathrm{J\,K^{-1}mol^{-1}})(298.15\,\mathrm{K}) \ln \left(4.1\times10^{-11}\right) \\ &= 59.3\,\mathrm{kJ/mol} \\ &= \Delta \bar{G}^{\circ}_{f(\mathrm{Ca^{2+}})} + 2\Delta \bar{G}^{\circ}_{f(\mathrm{F^{-}})} - \Delta \bar{G}^{\circ}_{f(\mathrm{CaF_{2}})}. \\ \therefore \Delta \bar{G}^{\circ}_{f(\mathrm{CaF_{2}})} &= \Delta \bar{G}^{\circ}_{f(\mathrm{Ca^{2+}})} + 2\Delta \bar{G}^{\circ}_{f(\mathrm{F^{-}})} - \Delta \bar{G}^{\circ} \\ &= -552.8 + 2(-281.52) - 59.3\,\mathrm{kJ/mol} = -1175\,\mathrm{kJ/mol}. \end{split}$$

3. (a) Half-reactions:

$$C_6H_{12}O_{6(aq)} \rightarrow 3C_2H_5OH_{(aq)}$$

 $C_6H_{12}O_{6(aq)} \rightarrow 6CO_{2(aq)}$

Balance for O:

$$\begin{array}{c} C_6H_{12}O_{6(aq)} \rightarrow 3C_2H_5OH_{(aq)} + 3H_2O_{(l)} \\ 6H_2O_{(l)} + C_6H_{12}O_{6(aq)} \rightarrow 6CO_{2(aq)} \end{array}$$

Balance for H:

$$\begin{aligned} 12H_{(aq)}^{+} + C_{6}H_{12}O_{6(aq)} &\rightarrow 3C_{2}H_{5}OH_{(aq)} + 3H_{2}O_{(l)} \\ 6H_{2}O_{(l)} + C_{6}H_{12}O_{6(aq)} &\rightarrow 6CO_{2(aq)} + 24H_{(aq)}^{+} \end{aligned}$$

Balance for charge:

$$\begin{split} 12H_{(aq)}^{+} + 12e^{-} + C_{6}H_{12}O_{6(aq)} &\rightarrow 3C_{2}H_{5}OH_{(aq)} + 3H_{2}O_{(l)} \\ 6H_{2}O_{(l)} + C_{6}H_{12}O_{6(aq)} &\rightarrow 6CO_{2(aq)} + 24H_{(aq)}^{+} + 24e^{-} \end{split}$$

Multiply the first reaction by 2 and add:

$$\begin{split} & 24H_{(aq)}^{+} + 24e^{-} + 2C_{6}H_{12}O_{6(aq)} + 6H_{2}O_{(l)} + C_{6}H_{12}O_{6(aq)} \\ & \to 6C_{2}H_{5}OH_{(aq)} + 6H_{2}O_{(l)} + 6CO_{2(aq)} + 24H_{(aq)}^{+} + 24e^{-} \\ & \therefore 3C_{6}H_{12}O_{6(aq)} \to 6C_{2}H_{5}OH_{(aq)} + 6CO_{2(aq)}, \quad \bar{n} = 24 \\ & \text{or } C_{6}H_{12}O_{6(aq)} \to 2C_{2}H_{5}OH_{(aq)} + 2CO_{2(aq)}, \quad \bar{n} = 8 \end{split}$$

(b)

$$\Delta \bar{G} = \Delta \bar{G}^{\circ} + RT \ln Q$$

$$= \Delta \bar{G}^{\circ} + RT \ln \left(\frac{(a_{C_{2}H_{5}OH})^{2}(a_{CO_{2}})^{2}}{a_{C_{6}H_{12}O_{6}}} \right).$$

$$\Delta \bar{G}^{\circ} = 2\Delta \bar{G}^{\circ}_{f(C_{2}H_{5}OH)} + 2\Delta \bar{G}^{\circ}_{f(CO_{2})} - \Delta \bar{G}^{\circ}_{f(C_{6}H_{12}O_{6})}$$

$$= 2(-181.64) + 2(-386.05) - (-910.23) \text{ kJ/mol} = -225.15 \text{ kJ/mol}.$$

$$\therefore \Delta \bar{G} = -225.15 \text{ kJ/mol}$$

$$+ (8.314472 \times 10^{-3} \text{ kJ K}^{-1} \text{mol}^{-1})(298.15 \text{ K}) \ln \left(\frac{(1.3)^{2}(0.03)^{2}}{0.7} \right)$$

$$= -240.35 \text{ kJ/mol}.$$

The maximum non-PV work is therefore 240.35 kJ/mol glucose.

(c)
$$\varepsilon = \frac{-\Delta \bar{G}}{\bar{n}\mathcal{F}} = \frac{240.35 \times 10^3 \,\mathrm{J/mol}}{8(96\,485.342\,\mathrm{C/mol})} = 0.311\,\mathrm{V}.$$

4. From the table of standard reduction potentials, we have

$$\therefore 2\text{Fe}_{(\text{aq})}^{3+} + 2\text{I}_{(\text{aq})}^{-} \to 2\text{Fe}_{(\text{aq})}^{2+} + \text{I}_{2(\text{s})}, \quad \varepsilon^{\circ} = 0.2355 \,\text{V} \quad \text{and } \bar{n} = 2.$$

$$\therefore \Delta \bar{G}^{\circ} = -\bar{n}\mathcal{F}\varepsilon^{\circ}$$

$$= -2(96\,485.342 \,\text{C/mol})(0.2355 \,\text{V}) = -4.544 \times 10^{4} \,\text{J/mol}.$$

$$\therefore K = e^{-\Delta \bar{G}^{\circ}/(RT)} = \exp\left(\frac{4.544 \times 10^{4} \,\text{J/mol}}{(8.314\,472 \,\text{J K}^{-1} \text{mol}^{-1})(298.15 \,\text{K})}\right)$$

 $Fe_{(aq)}^{3+} + e^{-} \rightarrow Fe_{(aq)}^{2+} +0.771 V$ $2I_{(aq)}^{-} \rightarrow I_{2(s)} + 2e^{-} -0.5355 V$

5. According to Raoult's law, $P_{\text{H}_2\text{O}} = X_{\text{H}_2\text{O}} P_{\text{H}_2\text{O}}^{\bullet}$, where $P_{\text{H}_2\text{O}}^{\bullet}$ is the vapor pressure of pure water.

$$X_{\rm H_2O} = \frac{54.56\,\mathrm{mol/L}}{54.56 + 3(0.4)\,\mathrm{mol/L}} = 0.978.$$

The factor of 3 in the calculation above comes from the fact that sodium sulfate dissociates into three ions in solution.

$$\therefore P_{\text{H}_2\text{O}} = (0.978)(149.38 \,\text{Torr}) = 146 \,\text{Torr}.$$

6. (a) For our reaction, $\Delta \bar{G} = \Delta \bar{G}^{\circ} + RT \ln(a_{H^{+}})^{\nu}$, since all other reactants and products are at unit activity. Since $\Delta \bar{G}^{\circ\prime} = \Delta \bar{G}$ at pH 7, we have

$$\Delta \bar{G}^{\circ\prime} = \Delta \bar{G}^{\circ} + \nu RT \ln(10^{-7})$$
$$= \Delta \bar{G}^{\circ} - 7\nu RT \ln 10.$$

(b) The formation reaction for a chloride ion would be

$$\frac{1}{2}Cl_{2(g)} + \frac{1}{2}H_{2(g)} \to Cl_{(aq)}^- + H_{(aq)}^+.$$

Thus $\nu = 1$, so that

$$\begin{split} \Delta \bar{G}^{\circ\prime} &= -131.218 \, \text{kJ/mol} - 7(1)(8.314472 \times 10^{-3} \, \text{kJ K}^{-1} \text{mol}^{-1})(298.15 \, \text{K}) \ln 10 \\ &= -171.174 \, \text{kJ/mol}. \end{split}$$

Bonus: HSO_4^- is a relatively strong acid. Its dissociation

$$HSO_4^- \rightleftharpoons H^+ + SO_4^-$$

would perturb the HSO_4^-/SO_3 equilibrium.