Chemistry 2720 Fall 2005 Quiz 6 Solution

1. Since $n = 1$ (from the note),
$$d = \frac{\lambda}{2\sin\theta} = \frac{2.079\,\text{Å}}{2\sin 75°} = 1.076\,\text{Å}.$$

2. $$p = \frac{h}{\lambda} = \frac{6.626\,068\,8 \times 10^{-34}\,\text{J/Hz}}{2.079 \times 10^{-10}\,\text{m}} = 3.187 \times 10^{-24}\,\text{kg m/s}.$$

3. According to the uncertainty principle,
$$\Delta p \geq \frac{h}{4\pi\Delta x}.$$
In this case, the uncertainty in the position after passing through the slit is 5 nm. (Our best guess about the position would be that the centre of the slit, so a neutron that gets through the slit could be as much as 5 nm away from our guess.) Thus,
$$\Delta p \geq \frac{6.626\,068\,8 \times 10^{-34}\,\text{J/Hz}}{4\pi(5 \times 10^{-9}\,\text{m})} = 1.055 \times 10^{-26}\,\text{kg m/s}.$$
This means that p (calculated in question 2) should sit between $p_{\text{min}} = 3.187 \times 10^{-24}\,\text{kg m/s} - 1.055 \times 10^{-26}\,\text{kg m/s} = 3.177 \times 10^{-24}\,\text{kg m/s}$ and $p_{\text{max}} = 3.187 \times 10^{-24}\,\text{kg m/s} + 1.055 \times 10^{-26}\,\text{kg m/s} = 3.198 \times 10^{-24}\,\text{kg m/s}$. Now calculate λ for these two momenta:
$$\begin{aligned}
\lambda_{\text{max}} &= \frac{h}{p_{\text{min}}} = \frac{6.626\,068\,8 \times 10^{-34}\,\text{J/Hz}}{3.177 \times 10^{-24}\,\text{kg m/s}} = 2.0859\,\text{Å}. \\
\lambda_{\text{min}} &= \frac{h}{p_{\text{max}}} = \frac{6.626\,068\,8 \times 10^{-34}\,\text{J/Hz}}{3.198 \times 10^{-24}\,\text{kg m/s}} = 2.0721\,\text{Å}. \\
\therefore \Delta\lambda &= \frac{1}{2}\left(2.0859 - 2.0721\,\text{Å}\right) = 0.007\,\text{Å}.
\end{aligned}$$
In case you're curious about the method based on advanced error analysis hinted at in the problem sheet, here it is:
$$\begin{aligned}
\lambda \pm \Delta\lambda &= \frac{h}{p \pm \Delta p} \\
&= \frac{h}{p} \pm \left|\frac{d}{dp}\left(\frac{h}{p}\right)\right| \Delta p \\
&= \frac{h}{p} \pm \frac{h}{p^2}\Delta p. \\
\therefore \Delta\lambda &= \frac{h}{p^2}\Delta p.
\end{aligned}$$
If you put in the numbers, you of course get the same answer as above. Note that there is no reason why you should have known this.