Chemistry 2720 Fall 2001 Assignment 9

Due: Tuesday, Dec. 4, 9:25 a.m.

This is the last assignment of the year. The final exam will be held Friday, Dec. 14 at 2:00 p.m. in room AH117. I will be keeping regular office hours during the exam period so you can drop in anytime to talk to me. Good luck with all your exams.

1. We established in class that quantum mechanical effects are not observable for large objects like instructors walking across the room or hockey pucks (although some goaltenders would swear otherwise). In this question, we will see whether we need to worry about quantum mechanics for things which are small, but not as small as atoms.

   A plant spore is a roughly spherical particle with a diameter of $1\,\mu$m and a density of $1\,\mathrm{g/cm^3}$. Small objects like these are subject to Brownian motion, which is to say that they tend to dance around due to collisions with molecules. Suppose that you wanted to measure the average speed due to Brownian motion of the spore described above. This can be done using a stage micrometer, essentially a glass slide with fine rulings etched into it (often by photolithography) at regular intervals. Stage micrometers with rulings which are $10\,\mu$m apart are commercially available at reasonable prices (in the neighborhood of 100 US$). The distances between the etched lines can be consistent to within $0.01\,\mu$m or better (depending on price). You plan to measure the speed of the spore by watching the spore move across a stage micrometer whose lines spacing is, as described above $10.00 \pm 0.01\,\mu$m. The speed is then just the distance between lines divided by the time taken to travel that distance. In the calculations which follow, assume that time can be measured much more accurately than position. The spore (and stage) is to be viewed through an optical microscope with a resolution of $0.5\,\mu$m.

   In one trial, it was found that the spore moved the distance between two rulings in 250 s.

   (a) What is the speed? [2 marks]

   (b) What is the uncertainty in the speed? [5 marks]

   (c) Does the uncertainty principle leave much room to improve the accuracy of this experiment? [10 marks]

2. An electron in a one-dimensional box has an absorption spectrum whose four longest-wavelength lines occur at 1104, 663, 473 and 414 nm. What is the length of the box? [10 marks]

   Hint: You will first need to identify the transitions, i.e. the initial and final values of the quantum number n. This is done partly by trial-and-error and partly by making one smart initial assignment.

3. Using a quantitative argument, show that a treatment of hydrogenic atoms which does not include relativistic effects, such as the de Broglie-Bohr theory, is inappropriate for U^{91+}. [5 marks]

   Notes: This is not a completely academic exercise. Hydrogenic uranium ions are used in a variety of experimental and theoretical scientific studies, both because they are interesting objects in themselves and because their study has some applications in materials science. Unfortunately, the relativistic theory of atoms is extremely complex so that our understanding of superheavy atoms is highly incomplete.

4. The Bohr theory of hydrogen had one singular success, namely the equation for the energy levels which led to the explanation of atomic spectra, and a number of failures. Briefly discuss one incorrect prediction made by the Bohr theory. [4 marks]

   Note: Limit yourself to one short paragraph and stick to the point. Excessively wordy answers may be penalized.