Appendix I

Properties of Exponential and Logarithmic Functions

1. $a^0 = 1$
2. $a^x a^y = a^{x+y}$
3. $a^x / a^y = a^{x-y}$
4. $a^{-x} = 1/a^x$
5. $(a^x)^y = a^{xy}$
6. $a^{1/x} = \sqrt[x]{a}$
7. $(ab)^x = a^x b^x$
8. Logarithms are inverse functions of exponentials:
   - $\log_a a^x = x$
   - $a^{\log_a x} = x$
9.
   - log is usually a shorthand notation for $\log_{10}$.[1] I generally prefer to explicitly write down the base, but feel free to use the shorthand if you prefer.
   - ln is called the "natural logarithm". $\ln x$ means the same thing as $\log_e x$, where $e = 2.71828\ldots$ is Napier's number. This very important logarithm comes up in many equations in chemistry. For our purposes, you mostly need to remember that it's just a logarithm that behaves like all other logarithms.

[1] One of the reasons that I like to explicitly write down the base is that some people, mostly mathematicians, use log for the natural logarithm rather than the base-10 logarithm. Writing down the base avoids any possible confusion.

10. $\log_a 1 = 0$
11. $\log_a(xy) = \log_a x + \log_a y$
12. $\log_a(x/y) = \log_a x - \log_a y$
13. $\log_a(1/x) = -\log_a x$
14. $\log_a(x^y) = y \log_a x$

Appendix J

Table of Integrals

This is not by any means a complete table of integrals. However, the integrals given here should be sufficient to work through the examples and exercises in this book.

1. $\int e^{ax}dx = \frac{1}{a}e^{ax}$
2. $\int x^n dx = \frac{1}{n+1}x^{n+1}$
3. $\int \frac{dx}{x} = \ln x$
4. $\int \frac{dx}{(a+bx)(c+ex)} = \frac{1}{ae-bc}\ln\left(\frac{c+ex}{a+bx}\right)$
5. $\int \frac{dx}{x^2(a+bx)} = \frac{b}{a^2}\ln\left(\frac{a+bx}{x}\right) - \frac{1}{ax}$
6. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a^2}\ln\left(\frac{x}{a+bx}\right) + \frac{1}{a(a+bx)}$
7. To integrate $\int dx\,(a+bx+cx^2)^{-1}$, first evaluate $q = 4ac-b^2$. Then,

$$\int \frac{dx}{a+bx+cx^2} = \begin{cases} \frac{2}{\sqrt{q}}\arctan\left(\frac{2cx+b}{\sqrt{q}}\right) & \text{if } q>0, \\ \frac{1}{\sqrt{-q}}\ln\left(\frac{2cx+b-\sqrt{-q}}{2cx+b+\sqrt{-q}}\right) & \text{if } q<0. \end{cases}$$