Chemistry 2000 Spring 2001 Section B Assignment 4 Solutions

1. (a) The solubility equilibrium is

$$Mg(OH)_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2OH^-_{(aq)}.$$

The solubility product (K_{sp}) of this salt is 1.5×10^{-11}. Thus

$$K_{sp} = (a_{Mg^{2+}})(a_{OH^-})^2.$$

Suppose that the solubility is s. Then $a_{Mg^{2+}} = s$ and $a_{OH^-} = 2s$.

$$\begin{aligned} \therefore 1.5 \times 10^{-11} &= s(2s)^2 = 4s^3. \\ \therefore s &= 1.55 \times 10^{-4}. \end{aligned}$$

The solubility is therefore $1.55 \times 10^{-4}\,\text{mol/L}$.

(b) In a pH 4 buffer, $a_{H^+} = 10^{-4}$. Using K_w, we find that $a_{OH^-} = 10^{-10}$. Since $a_{Mg^{2+}} = K_{sp}/a^2_{OH^-}$, we find a solubility of $1.5 \times 10^9\,\text{mol/L}$. This absurdly high number indicates that the solubility will be extremely high, and mostly limited by the buffering capacity of the solution.

2. We can use the molar mass to convert the solubility to mol/L:

$$s = \frac{0.02\,\text{g/L}}{126.71\,\text{g/mol}} = 1.6 \times 10^{-4}\,\text{mol/L}.$$

The solubility process is

$$GaF_{3(s)} \rightleftharpoons Ga^{3+}_{(aq)} + 3F^-_{(aq)}.$$

The concentration of gallium in solution will be equal to s while the concentration of fluoride will be $3s$. Accordingly,

$$K_{sp} = s(3s)^3 = 27s^4 = 1.7 \times 10^{-14}.$$

3. (a) 1 kg of maple sugar is 2% of the initial weight of sap, i.e. $0.02m = 1\,\text{kg}$, which implies that $m = 50\,\text{kg}$.

(b) We need to take the sap from 5°C to the boiling point, assumed to be about 100°C.[1] The amount of heat required to do this is

$$q_1 = m\tilde{C}_P\Delta T = (50\times 10^3\,\text{g})(4.18\,\text{J}\,\text{K}^{-1}\text{g}^{-1})(95\,\text{K}) = 20\,\text{MJ}.$$

We must also boil away 49 kg of water. The heat of vaporization of water is 2260 J/g, so that takes

$$q_2 = (49\times 10^3\,\text{g})(2260\,\text{J/g}) = 111\,\text{MJ}.$$

The total heat required is therefore 131 MJ.

(c) $m_{\text{wood}} = (131\,\text{MJ})/(4\,\text{MJ/kg}) = 33\,\text{kg}$.

4. (a) We need a balanced reaction:

$$\text{Na}_{(s)} + \text{H}_2\text{O}_{(l)} \rightarrow \text{NaOH}_{(aq)} + \frac{1}{2}\text{H}_{2(g)}.$$

For this reaction,

$$\begin{aligned}\Delta\bar{H}^\circ &= \Delta\bar{H}^\circ_{f(\text{NaOH}_{(aq)})} + \frac{1}{2}\Delta\bar{H}^\circ_{f(\text{H}_{2(g)})} - \left(\Delta\bar{H}^\circ_{f(\text{Na}_{(s)})} + \Delta\bar{H}^\circ_{f(\text{H}_2\text{O}_{(l)})}\right)\\ &= -470.114 + \frac{1}{2}(0) - [0 + (-285.830)]\ \text{kJ/mol}\\ &= -184.284\,\text{kJ/mol}.\end{aligned}$$

The number of moles of sodium is

$$m_{\text{Na}} = \frac{10\,\text{g}}{22.9898\,\text{g/mol}} = 0.43\,\text{mol}.$$

The total heat released is therefore

$$q = (-184.284\,\text{kJ/mol})(0.43\,\text{mol}) = -80\,\text{kJ}.$$

(b) The reaction is exothermic so the water *gains* 80 kJ of heat.

$$\Delta T = \frac{80\times 10^3\,\text{J}}{(400\,\text{g})(4.18\,\text{J}\,\text{K}^{-1}\text{g}^{-1})} = 48\,\text{K}.$$

The final temperature would be $15 + 48$°C $= 63$°C.

(c) The reaction produces 0.43 mol of hydroxide ions. The volume of the solution is $(400\,\text{g})/(0.9971\,\text{g/mL}) = 401\,\text{mL}$. The concentration of hydroxide is therefore $(0.43\,\text{mol})/(0.401\,\text{L}) = 1.1\,\text{mol/L}$. The activity of hydrogen ions would be approximately $10^{-14}/1.1 = 9.2\times 10^{-15}$, which gives a pH of 14.0.

[1] This is probably the shakiest assumption in the whole calculation since the boiling point will increase as the sap becomes more concentrated.