Chemistry 1000 Lecture 3:
Nuclear stability

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Radioactive decay series

Forces between nucleons

Electrostatic (Coulomb) force:
- Repulsive force between protons
- Neutrons not involved
- Decreases with distance as $1/r^2$

Strong nuclear force:
- Attractive force between nucleons (neutrons and protons)
- Decreases exponentially with distance (for nucleons)
Consequences:

- Large nuclei unstable because strong nuclear force decreases in strength with distance faster than Coulomb force
  Heaviest stable nucleus: $^{208}\text{Pb}$
- Effect of neutrons: strong-force stabilization and increase in average distance between protons \(\therefore\) less Coulomb repulsion
- Neutrons and protons are fermions, so they obey the Pauli exclusion principle (to be studied later).
  Putting more nucleons into a nucleus forces the nucleons into higher energy states.
- Having too many neutrons (requiring the use of higher nuclear energy levels) tends to result in beta decay
Nuclear stability rules

Notation: $N = \text{number of neutrons} = A - Z$

- For small $Z (< 20)$, $N \approx Z$ for stable nuclei.
  Example: Carbon has two stable isotopes, $^{12}_6\text{C}$ (98.9%) and $^{13}_6\text{C}$ (1.1%).

- For larger $Z$, $N > Z$, with the $N/Z$ ratio rising slowly from 1 to 1.54 as $Z$ increases from 20 to 82.

- No stable nuclei for $Z > 82$ ($^{208}_{82}\text{Pb}$)

- Nuclei with even numbers of neutrons or even numbers of protons or, better still, both, are more likely to be stable

Examples:
- Stable nuclei of iron ($Z = 26$): $^{54}_{26}\text{Fe}$ ($N = 28$), $^{56}_{26}\text{Fe}$ ($N = 30$),
  $^{57}_{26}\text{Fe}$ ($N = 31$), $^{58}_{26}\text{Fe}$ ($N = 32$)
- Cobalt has just one stable nucleus: $^{59}_{27}\text{Co}$ ($N = 32$)
Region of nuclear stability

Nuclear decay series explained

- Too many neutrons?
  - Beta decay
- Too few neutrons?
  - Positron emission, or
  - Electron capture, or
  - Alpha emission (esp. for very heavy nuclei)
- Too heavy?
  - Alpha decay
    - especially favored for even proton/even neutron nuclei since alpha decay maintains this favorable parity
Nuclear decay series explained
Examples from the decay series of $^{232}_{90}$Th

$^{232}_{90}$Th has $N/Z = 142/90 = 1.58 > 1.54$, so too many neutrons, but it’s also very heavy ($232 > 208$). It has even numbers of both neutrons and protons, so alpha decay would maintain this favorable parity
⇒ alpha decay to $^{228}_{88}$Ra

$^{228}_{88}$Ra has $N/Z = 140/88 = 1.59$, so the preceding alpha decay has made the neutron excess worse
⇒ beta decay to $^{228}_{89}$Ac ($N/Z = 1.58$)

$^{228}_{89}$Ac still has too many neutrons and an odd number of protons
⇒ beta decay to $^{228}_{90}$Th ($N/Z = 1.53$)
Nuclear decay series explained
Further examples

\[ \frac{18}{9} F \] has \( N/Z = 1 \) (good) but odd numbers of both protons and neutrons (bad)
\[ \Rightarrow \] positron emission to form \( \frac{18}{8} O \)

\[ \frac{84}{40} Zr \] has \( N/Z = 1.1 \), which turns out to be below the region of stability (i.e. \( N \) is too small)
\[ \Rightarrow \] electron capture
Nuclear binding energy

- Imagine taking initially separated protons, neutrons and electrons and assembling them into an atom.
- This process would be massively exothermic, largely because of the nuclear binding energy, $\Delta E$ for this process.
- The nuclear binding energy per nucleon is

$$E_b = \Delta E/A$$

- Important: This is a calculation you do using the mass of a particular isotope, not the average atomic mass.
Nuclear binding energy

Example: $^{62}_{28}\text{Ni}$

$$28^1_1p + 34^1_0n + 28e \longrightarrow ^{62}_{28}\text{Ni}$$

$$m(^{62}_{28}\text{Ni}) = 61.928\, 3451\, \text{u} \equiv 61.928\, 3451\, \text{g mol}^{-1}$$

$$= \frac{61.928\, 3451\, \text{g mol}^{-1}}{(1000\, \text{g kg}^{-1})(6.022\, 141\, 29 \times 10^{23}\, \text{mol}^{-1})}$$

$$= 1.028\, 344\, 27 \times 10^{-25}\, \text{kg}$$

$$\Delta m = m(^{62}_{28}\text{Ni}) - (28m_p + 34m_n + 28m_e)$$

$$= 1.028\, 344\, 27 \times 10^{-25}$$

$$- [28(1.672\, 621\, 78 \times 10^{-27}) + 34(1.674\, 927\, 35 \times 10^{-27})$$

$$+ 28(9.109\, 3829 \times 10^{-31}\, \text{kg})]$$

$$= -9.7202 \times 10^{-28}\, \text{kg}$$
Nuclear binding energy
Example: $^{62}_{28}$Ni (continued)

\[
\Delta E = \Delta mc^2 \\
= (-9.7202 \times 10^{-28} \text{ kg})(2.99792458 \times 10^8 \text{ m/s})^2 \\
= -8.7361 \times 10^{-11} \text{ J}
\]

\[\therefore E_b = \Delta E / A\]

\[= (-8.7361 \times 10^{-11} \text{ J})/62\]

\[= -1.4090 \times 10^{-12} \text{ J}\]
Nuclear binding energy

Binding energy per nucleon

$E_b / 10^{11} \text{ J mol}^{-1}$

$A$
Some unusually stable nuclei

- $^4_2\text{He}$: $E_b = -6.83 \times 10^{11} \text{ J mol}^{-1}$ compared to (e.g.) $-5.14 \times 10^{11} \text{ J mol}^{-1}$ for $^6_3\text{Li}$. \\
  $\Rightarrow$ explains alpha particle emission as a common nuclear decay process (rather than some other fragment)

- Minimum of binding energy curve (strongest binding) is $^{62}_{28}\text{Ni}$: $-8.49 \times 10^{11} \text{ J mol}^{-1}$ [not $^{56}_{26}\text{Fe}$ at $-8.48 \times 10^{11} \text{ J mol}^{-1}$]
  $\Rightarrow$ minimum near these nuclei explains abundance of metals from this part of the periodic table
Fission vs fusion

- To the left of the minimum, fusion can form nuclei closer to the minimum, so this process tends to be energetically favorable.
- To the right of the minimum, fission can form more tightly bound nuclei, so it tends to be energetically favorable.
- Steepest part of curve is at low $A$, so fusion of light nuclei generates more energy as a rule.