# Estimating Inequality in the Distribution of Welfare Using Demand Models[1]

by

Christopher J. Nicol[2]
Department of Economics
University of Regina
Regina, Saskatchewan, S4S 0A2
CANADA
306-585-4182; FAX 306-585-4815
Internet: Chris.Nicol@uregina.ca

January 20, 1997

[1]This research was supported in part by grants from the Social Sciences and Humanities Research Council of Canada, and the University of Regina President's Research Fund. Earlier versions of this paper were presented at the 1990 Meetings of the Canadian Econometric Study Group, University of Guelph, Ontario, and the 1991 European Meetings of the Econometric Society, Cambridge, England. I thank Charles Beach, Ian Rongve, David Ryan, Jason Walters and two anonymous referees for helpful comments. Brian Southam provided excellent research assistance. Any remaining errors are my responsibility.

[2]Department of Economics, University of Regina.

**Abstract**

In this paper two demand models with general household equivalence scales (GES) are estimated. These GES are identifiable, since they have the independence of base utility (IB) or equivalence scales exactness (ESE) property. Estimates of household characteristics adjusted income can then be calculated relative to a specific household type. This "individual equivalent income (IEI) is then used to calculate measures of inequality in the distribution of welfare. As more than one model is estimated, the sensitivity of these estimates to model specification changes can be considered. Comparisons are also made to estimates of inequality based on household income.

It is found that absolute inequality is sensitive to model specification, but relative inequality is not. This is true using indices of inequality of the Atkinson (1970), Kolm (1976a,b) and Sen (1973) type, or more general measures of inequality based on estimates of Lorenz curve decile ordinates, constructed using methods introduced by Beach and Davidson(1983). Thus, if absolute measures of inequality are required, these results suggest some further research is required to determine a class of models which is less sensitive to model specification error. However, if only relative measures of inequality are needed, then estimates of inequality appear to be very robust to the choice of model specification.

**Keywords:**
equivalence scales; welfare inequality; translog; almost ideal demand system.

## Section I: Introduction

The measurement of inequality is a fundamentally difficult problem for a variety of reasons. A simplistic approach would be to use the distribution of household incomes to measure inequality based on a summary statistic such as the Gini coefficient. However, household incomes do not account for differences in household composition, and Gini coefficients are difficult to compare when the Lorenz curves on which they are based intersect. A recent paper by Davies and Hoy (1995), however, makes the latter difficulty relatively easier to deal with. This requires that one be willing to accept the normative axiom of *aversion to downside inequality* (ADI), in addition to the *principle of transfers*, embodied in the Lorenz criterion. The Lorenz criterion assumes only that a transfer from richer to poorer reduces inequality (Davies and Hoy, 1995, p. 980).

The former problem of differences in household characteristics remains. This can be dealt with by somehow adjusting household income so that it reflects differences in household characteristics. For example, general household equivalence scales (GES)[3] from an estimated demand model could be used to rescale the actual income levels of households. However, there are difficulties in the application of this approach. Although the estimation of demand functions does not require interpersonal comparisons of utility levels, estimation of GES does.

Interpersonal comparisons of utility levels requires an additional layer of information to that which is required for demand system estimation. If the former kind of information is available, then this information structure is defined as *ordinal full comparability* (OFC) by Blackorby and Donaldson (1993a). Then, if the household cost function is given by $C[u, p, A]$, where $u$ is utility; $p$ a price vector for the goods consumed; and $A$ is the household's characteristics vector, a GES can be defined by the function $D[u, p, A] = C[u, p, A]/C[u, p, A^r]$, where $A^r$ is the household characteristics vector for a reference household (Blackborby and Donaldson, 1993a).

Lewbel (1989a) and Blackorby and Donaldson (1993a) explain that the foregoing type of information structure is not required to recover GES from demand system estimation, if $C[u, p, A]$ can be written $\bar{C}[u, p] \cdot \hat{C}[p, A]$, for all $u, p$ and $A$. This condition is referred to by Lewbel (1989a) as the IB property (Independence of Base level of income or utility), and by Blackorby and Donaldson (1993a) as ESE (equivalence scale exactness).

Given an indirect utility function (and its dual cost function) dependent on household characteristics, one can obtain the associated demand system. The parameters of these utility and

[3]Equivalence scales can be of the Engel (general household equivalence scales) or Barten (commodity-specific household equivalence scales) type. For the purposes of estimating household characteristics adjusted income, however, both types of scale reduce to an adjustment factor, depending on the households' characteristics.

demand functions can then be used to define commodity-specific equivalence scales (CES), or GES. There are, however, an infinite set of equivalence scales that could be defined in this way, and still be consistent with the specified demand system if it were estimated. On the other hand, certain restrictions can be placed on the parameters of the GES (these parameters being parameters of the underlying demand and utility functions), and a unique set of GES obtained. The natural kinds of restrictions to use in this setting are those which make the GES have the IB/ESE property, since the GES will then be independent of utility in such a parametrisation. These restrictions can be interpreted as identifying restrictions for the GES. Some of these identifying restrictions are empirically testable. If they are rejected, then the associated GES cannot be viewed as an appropriate representation of the data, for the purposes of comparing households with different characteristics.

For a given demand system, it is therefore possible to impose restrictions which yield GES with the IB/ESE property. This circumvents the dependence of the GES on utility by imposing restrictions which essentially select a particular GES out of the infinite number which a set of demand data and a given demand system would rationalise. Consequently, this approach does not avoid the non-identifiability of GES in general, it merely indicates the set of identifiability restrictions to be used in order to yield a unique set of GES.

Since the parameters of certain demand models can be used to obtain IB/ESE GES, these GES can in turn be used to obtain household characteristics scaled income. Lewbel (1993) indicates (in response to some observations in Blackorby and Donaldson (1993b)), that this scaled income can be used with classes of social welfare functions based on certain indirect utility functions, to compare the aggregate welfare associated with different income distributions. Two indirect utility functions suitable for this purpose, which are widely used empirically, include the almost ideal demand system (AIDS), Deaton and Muellbauer (1980), and the translog model of aggregate consumer behaviour, Jorgenson, Lau and Stoker (1982) (JLS).

Given a vector of household characteristics scaled income, summary statistics of inequality in the distribution of this income can be calculated. Indices of the Atkinson (1970), Kolm (1976a,b) and Sen (1973) (AKS indices) type can be used for this purpose. Different values of the "inequality aversion parameter" will yield different values for this index, which will indicate how inequality is affected when individuals in the lower end of the distribution receive relatively higher or lower weight in the calculation of the AKS index. Alternatively, Lorenz curve ordinate vectors and their covariance matrices can be calculated, as suggested by Beach and Davidson (1983). These vectors can then be compared for different income distributions, and tested for statistically significant differences. Also, for Lorenz curves which intersect, households in the same ranges of different income distributions receiving statistically significantly more or less income can be determined.

To construct the foregoing types of inequality measure, it is therefore necessary, as a first step, to specify appropriate models of consumer behaviour. In this context, "appropriate" has the meaning that the indirect utility function(s) specified must yield demand systems with IB/ESE GES. The JLS is a model of this type. Jorgenson and Slesnick (1987) and Nicol (1994) have recently used this model to estimate CES and GES. The former study was based on United States data, the latter on Canadian data. In this paper, the GES in Nicol (1994) are used to construct household characteristics scaled income.

As explained earlier, however, even for a given demand model, there is an infinite set of GES, depending on the identifying restrictions used to estimate the GES. Imposing exact aggregation on a translog demand system ensures that the associated GES for the system have the required IB/ESE property. In addition, there are other demand models with IB/ESE GES which could be used.

For comparative purposes, an AIDS model incorporating IB/ESE GES is also estimated. The IB/ESE GES from the JLS and AIDS models are used to compute household characteristics adjusted, or "individual equivalent" income (IEI). IEI is based on the "equally distributed equivalent" levels of income proposed in Atkinson (1970), and used in Kwong (1987) with the JLS model. Jorgenson and Slesnick (1984) used a slightly different method to estimate inequality for the United States.

Using these different IB/ESE GES, and several parameterisations of the JLS and AIDS models, alternative estimates of IEI were calculated. This allows a sensitivity analysis to be conducted, to determine how different measures of IEI affect inequality defined in terms of AKS indices, and Lorenz curve ordinates. It is also possible to compare these measures of inequality with similar, naive measures based on *actual* income, unadjusted for differences in household characteristics.

Several interesting results were obtained. Increased model restrictiveness implied a lower apparent degree of absolute inequality. This was true whether inequality was measured using AKS indices or IEI quantiles. More specifically, the inequality indices showed that the degree of inequality in IEI rose over the period 1978–1986, irrespective of the estimated model used. This feature was even more apparent when looking at IEI quantiles. Low income households exhibited a decreasing share of IEI from 1978–1986, no matter which model the estimates were based on. Only high income households (above the seventh decile) exhibited a rising share of IEI over time.

Analysis of inequality in IEI for all households together pre-supposes that the conversion of household income into IEI using equivalence scales handles the differences in household characteristics adequately, and also that such conversion is indeed necessary. It is possible to analyse these issues by looking at inequality in the distribution of actual household income (with and without adjustments for differences in household characteristics), and in the distribution of IEI for different household sizes.

Inequality in actual household income rose from 1978–86, but not to a statistically significant extent, based on the AKS indices. This was borne out when considering inequality in household income for most of the individual household sizes. However, absolute inequality was lower using actual income, than when using IEI.

Inequality based on IEI (when compared for specific family sizes across time) did *not* exhibit the same pattern as IEI inequality or actual income inequality for all households together. However, IEI inequality for households of differing sizes need not yield the same pattern of inequality as comparing IEI for groups of households of the same size, since IEI adjustments allow for differences in household characteristics other than family size. Comparing IEI for households of a given size is therefore not necessarily appropriate. The important point is that the pattern of *relative* inequality based on IEI for all households; and on actual income for households stratified by family size was the same, although *absolute* inequality appeared higher for the latter.

The results therefore show that there are variations in the outcomes observed which depend, to a degree, on the model specification used. Some recent research offers conflicting evidence on the appropriate rank of demand system which should be used when employing microdata (See, for example, Lewbel, 1991, Blundell, Pashardes and Weber, 1993, Banks, Blundell and Lewbel, 1994, and Nicol, 1995a,b and 1996). Further research on this and a number of other model specification issues having implications for the estimation of GES and IEI is ongoing.

The remainder of the paper is structured as follows. In Section 2, the empirical models used to estimate GES and thus IEI are presented. Section 3 describes the data, estimation and results. AKS indices of inequality are presented and compared, as are estimated Lorenz curve ordinate vectors and their standard errors. Some test results for differences in the estimated Lorenz curves are also presented and discussed. Section 4 summarises and concludes.

**Section II: Household Demands, Identifiable Equivalence Scales, Equivalent Income and Inequality Measures**

*Section II.1: Demand Systems With Identifiable General Household Equivalence Scales*

Lewbel (1989a) showed that the GES in the JLS model have the IB/ESE property, owing to the imposition of exact aggregation restrictions. These restrictions mean the GES depend only on $p$ and $A$. This can be seen by considering the JLS indirect utility function,

$$\ln V_k = \alpha_0 + \{\ln \frac{p}{M_k}\}^T \alpha_p + \frac{1}{2}\{\ln \frac{p}{M_k}\}^T B_{pp} \ln \frac{p}{M_k} + \{\ln \frac{p}{M_k}\}^T B_{pA} A_k \tag{1}$$

where $\ln\{p/M_k\}$ is a vector of logs of normalised prices, $\{p_1, \ldots, p_N\}$, deflated by total expenditures, $M_k$, on $N$ goods; $A_k$ is the vector of the $k$'th household's characteristics; and $\alpha_p, B_{pp}$ and $B_{pA}$ matrices of parameters. Exact aggregation restrictions in income, $\iota^T B_{pp}\iota = 0$, and household characteristics, $\iota^T B_{pA} = 0$, yield GES ($m_{k0} = M_k[p, A_k]/M_0[p, A_0]$) with the IB/ESE property.

The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) is also exactly aggregable in income. The basic AIDS can be modified to incorporate household characteristics effects in a variety of ways. (See Lewbel, 1985, for a detailed exposition of the ways in which this can be achieved, in a unified framework.) Modifying the AIDS to include household characteristics effects which yields a system which is still exactly aggregable in income and, now, household characteristics, leads to an alternative to the JLS system, which still incorporates IB/ESE GES. The modified AIDS model used in this paper is obtained from the log expenditure function

$$\ln M_k[u_k, p, A_k] = \ln[a(p) \cdot n(A_k)] + u_k \cdot b(p) \tag{2}$$

where the functions $a(p), b(p)$ and $n(A_k)$ are

$$\ln a(p) = \alpha_0 + \ln p^T \alpha + \frac{1}{2} \ln p^T \Gamma \ln p \tag{3}$$

$$b(p) = \beta_0 \prod_i^N p_i^{\beta_i} \tag{4}$$

$$\ln n(A_k) = \delta^T A_k \tag{5}$$

*Section II.2: Individual Equivalent Income*

IEI is defined as the income level, $M_k^e$, which equates

$$\ln V(p^0, M_k^e, A_0) = \ln V(p^k, M_k, A_k) \tag{6}$$

Given the JLS and AIDS systems specified above, which incorporate IB/ESE GES, it is possible to obtain expressions for IEI, $M_k^e$, which depend on these GES. Since the only elements in these measures of $M_k^e$ which can potentially depend on utility are the GES, using demand models which have IB/ESE GES means that the expressions for $M_k^e$ will not depend on utility. The alternative expressions for IEI based on the different models can therefore be viewed as embodying alternative identifiability restrictions which yield unique estimates of IEI. These unique estimates are, of course, only some out of the infinite set which could be calculated, depending on the GES identifiability restrictions which are introduced.

The IEI obtained by this method therefore represent income levels which have been adjusted to account for differences in household characteristics, and in the differences in prices which households face. The reference level of utility implicit in the definition of IEI in (6), $V_k^0$, however, does not enter explicitly into the computation of these estimates.

*Section II.3: Inequality Measures Based on Individual Equivalent Incomes*

In this study, AKS indices of inequality and IEI decile ordinates are calculated for the JLS and AIDS models, as discussed in Section III below. The AKS index of inequality, $\mathcal{I}$, depends on IEI as follows

$$\mathcal{I} = 1 - \frac{\Xi}{\mathcal{M}} = 1 - \frac{W^*(M_1^e, \ldots, M_N^e)}{1/N \sum_k^N M_k^e} \ , 0 \leq \mathcal{I} \leq \ 1 \tag{7}$$

As $\mathcal{I}$ increases, inequality in the distribution of IEI increases. If $\mathcal{I} = 0$, the actual distribution of IEI, summarised by $W^*(M_1^e, \ldots, M_N^e)$, is equal to mean equivalent income, $\mathcal{M}$. If $W^*(M_1^e, \ldots, M_N^e)$, is a social welfare function, then $\mathcal{I}$ measures inequality in the distribution of social welfare. Blackorby and Donaldson (1982) have discussed the properties of the mean of order $r$ parameterisation of $W^*(M_1^e, \ldots, M_N^e)$, which can be written

$$W^*(M_1^e, \ldots, M_N^e) = [1/N \sum_j^N [M_j^e)^r]^{(1/r)} \ , r \neq 0 \tag{8}$$

or

$$W^*(M_1^e, \ldots, M_N^e) = \prod_j^N [M_j^e]^{1/N} \ , r = 0 \tag{9}$$

They state that, as $r \to -\infty$, $W^*(M_1^e, \ldots, M_N^e)$ becomes more egalitarian, whereas as $r \to 1$, $W^*(M_1^e, \ldots, M_N^e)$ becomes more utilitarian. In this application, $\mathcal{I}$ is computed for $r = -2, -1, 0$ and 0.5.

AKS indices conditional on different values of $r$ provide summary measures of inequality based on differing normative perspectives. A more detailed picture of inequality can be obtained, however, by estimating Lorenz curve ordinates for IEI. Vectors of these ordinates and their covariance matrices can be estimated consistently using the methodology proposed in Beach and Davidson

(1983). The $i$'th IEI decile ordinate, $\hat{\Phi}_i$, is estimated as follows

$$\hat{\Phi}_i = d_i \cdot \frac{\hat{\tau}_i}{\hat{\mathcal{M}}}, \qquad\qquad i = 1, \ldots, 9 \tag{10}$$

where $d_1, \ldots, d_9 = 0.10, \ldots, 0.90$; $\hat{\tau}_i = \sum_{k=1}^{r_i} M^e_{(k)}/r_i$; $r_i = Nd_i$; $\hat{\mathcal{M}} = (1/N)\sum_{k=1}^{N} M^e_{(k)}$; $M^e_{(k)}$ is sample IEI sorted from lowest to highest; and $N$ is the sample size. Also, $\hat{\tau}_i$ and $\hat{\mathcal{M}}$ are conditional and unconditional mean IEI respectively. The former is conditional on the IEI falling in the $i$'th decile.

Beach and Davidson (1983), Theorem 2, show that $\hat{\Phi} = [\hat{\Phi}_1, \ldots, \hat{\Phi}_9]^T$ is asymptotically normal, and that $\sqrt{N}(\hat{\Phi} - \Phi) \stackrel{A}{\sim} \mathrm{N}(0, V_L)$. Given IEI estimates for several years, Lorenz curve decile vectors and their variance-covariance matrices can be estimated for those years. Whether IEI is changing statistically significantly over time can then be tested generally. Alternatively, tests for differences in particular deciles across time can be be conducted. Test results of both types are presented in Section III below.

## Section III: Data and Results

### *Section III.1: Introduction*

Parameter estimates from Nicol (1994) for the JLS model with and without homogeneity and symmetry imposed were used to calculate alternative series of IEI estimates in 1978, 1982, 1984 and 1986. The calculation of IEI was computed for households drawn from the 1978, 1982, 1984, and 1986 *Survey of Family Expenditures Microdata Files (FAMEX)* for Canada. These were the households on which the Nicol (1994) JLS estimation was based. That is, married couple families with varying numbers of children, excluding the self-employed. Earlier research indicates that household tenure status is an important determinant of demand behaviour (Barnes and Gillingham, 1984; Nicol, 1989a). To recognise this, and to avoid the need to deal with expenditures on housing, the households include only those living in rented accommodation. Furthermore, the FAMEX masks high income households, and does not adequately cover certain types of low income households. These points should be borne in mind when interpreting the resulting inequality measures.

The household characteristics which were included in the Nicol (1994) demand model result in adjustment to household income to yield the IEI estimates. These characteristics were family size; age of head; region of residence; and urban versus rural resident. Definitions of these categories are given in Table 1. As can be seen, the households have a wide range of characteristics. The IB/ESE GES resulting from this kind of setting therefore deals with a lot more than just family size, which is usually viewed as the most important characteristic to focus on when estimating GES.

However, the other household characteristics in Table 1 were all found to be statistically significant determinants of demand in Nicol (1994) (and in other research: see, for example, Nicol, 1989a and 1991). These variables are therefore important determinants of the IB/ESE GES functions.

Although a wide range of household characteristics are modelled, requiring a large number of parameters to be estimated via the demand model, the total number of households in the surveys which could be used is very large. That is, to compute IEI for 1978, 1982, 1984 and 1986, a total of 8933 households were taken from the four FAMEX surveys. The breakdown by survey year was 2163, 2700, 1436 and 2634 observations respectively. This is advantageous for the computation of standard errors for the AKS indices, and of decile ordinate vectors and their variance-covariance matrix estimates, since such estimates are only valid asymptotically.

The (GES-adjusted) AIDS model was not estimated in Nicol (1994), but was estimated for this study, using the same data as Nicol (1994). While the JLS and AIDS models are non-nested, the former has many more parameters than the latter. An informal comparison of the estimated log-likelihoods for the two models indicated that the AIDS would be strongly rejected in favour of the JLS model, if a suitable encompassing model were to be estimated[4]. As in the JLS case, the homogeneity and symmetry restrictions were strongly rejected for the AIDS model with GES. These homogeneity and symmetry test results can be interpreted as model specification tests (Keuzenkamp and Barten, 1991). This therefore indicates that both models leave something to be desired. Nonetheless, these models are widely used in applied demand analyses, although there has recently been interest in modelling demand systems with more complicated Engel functions. See, for example, the rank three system estimated by Blundell, Pashardes and Weber (1993)[5]. These authors find evidence that quadratic terms in functions of real expenditures are important deter-

[4]There is an extensive literature on the relationship between the JLS (translog) and AIDS models. Lewbel (1989b) and Nicol (1989b) looked at this from slightly different perspectives. Formal statistical comparisons of the two models have been made using Lewbel's (1989b) more general model, which has the JLS and AIDS models as special cases. Wang, Halbrendt and Johnson (1996) compare the two models directly using an asymptotically valid, non-nested test procedure defined in Vuong (1989), and cite a number of other comparisons in this literature, which used an encompassing approach. On balance, this literature indicates that neither the JLS nor AIDS models are unambiguously preferable. That is, some of these studies find the JLS superior (as here), for example, Bollino and Violi (1990); whereas others find the reverse, for example, Ramezani, Rose and Murphy (1995). These studies are noteworthy since they use cross-sectional microdata. Wang, Halbrendt and Johnson (1996), on the other hand, find the models to be indistinguishable, but they use aggregated data, a very small sample and a very limited range of goods in their demand system (cigarettes, liquor, beer and tea).

[5]Lewbel (1991) defines the rank of any demand system as the maximum dimension of the function space spanned by the Engel curves of the demand system.

minants of demand. However, the importance of these terms is not by any means a settled issue, since they were found to be statistically insignificant in some more recent work (Nicol, 1995a,b and 1996), and they were found to be unnecessary in some equations of the demand system estimated by Banks, Blundell and Lewbel (1994). This last study, however, is of some interest in the present context, since welfare calculations were found to be sensitive to the exclusion of quadratic expenditure terms from *all* equations of the demand system. The rejection of homogeneity and symmetry by the models estimated in this paper, indicates that some alternative model specifications could fruitfully be explored. Rank three systems are therefore an obvious area for such work, and this is currently the subject of research following up this work. In this paper, the sensitivity of the inequality measures analysed when the model specification is changed provides a useful guide to the robustness of such estimates to model specification.

*Section III.2: AKS Indices and IEI/Income Distribution Summaries*

As indicated earlier, the homogeneity and symmetry restrictions were rejected for both the JLS and AIDS models on which estimates of IEI are based. To assess how this affects the resulting estimates of inequality, estimates based on both models with and without homogeneity and symmetry imposed are presented.

Tables 2 and 3 contain estimates of $\mathcal{I}$ conditional on the JLS model with homogeneity and symmetry both imposed and relaxed respectively. According to Table 2 (homogeneity and symmetry imposed), inequality in the distribution of IEI rises from 1978–1986 for all values of $r$ except -2.0. In any given year, inequality appears greater the higher the value of $r$. This indicates that inequality is increasing particularly for households in the low end of the IEI distribution. When these households receive less weight in the calculation of the AKS index (as the value of $r$ declines), so the appearance of inequality increases. This interpretation is supported by the Table 3 results. However, it should also be noted that the estimated AKS indices are qualitatively different for corresponding elements in Tables 2 and 3. The values in the latter Table are implausible. The source of these qualitatively different values, as seen in Table 3, arises from the imprecision in estimation of parameters attached to the price variables in the JLS model. When homogeneity and symmetry are relaxed, the increased variability in the parameter estimates on price variables results in IEI estimates for a small group of households being extremely large, thereby giving the impression that these households receive most of the IEI in the total distribution. This emphasises that the existence of the cost function and indirect utility function depend on the imposition of homogeneity and symmetry. When these are not imposed, the interpretation of the IEI becomes suspect, which leads to the AKS indices in Table 3 having the pattern shown.

Tables 4 and 5 contain the estimated AKS indices conditional on the AIDS model with homogeneity and symmetry imposed and relaxed respectively. Again, inequality increases from 1978–1986, except when $r = -2.0$. In addition, inequality in any given year is higher the lower is $r$. These results are qualitatively the same as in Tables 2 and 3. It is also true that absolute inequality appears worse when the homogeneity and symmetry restrictions are relaxed ($\mathcal{I}$ values are higher).

The foregoing analysis is highlighted further when plotting the densities associated with the different variants of IEI, for different model specifications, and for different years. Figures 1 and 2 give plots of the estimated IEI distributions for 1978 and 1986, for the homogeneity and symmetry imposed and relaxed JLS models respectively. These distributions were estimated using a nonparametric kernel method (see Härdle, 1990, for a detailed exposition of this methodology), and the scales of the plots adjusted to highlight the behaviour of the respective distributions. As mentioned above, the distribution of IEI with the restrictions relaxed (Figure 2) is very uneven, with most of IEI being concentrated in a few households. This is captured by the high values of the AKS indices presented in Table 2, as discussed above. It also appears to be the case that IEI, based on estimation of the JLS model, is unimodal. However, this is not the case when looking at the corresponding estimated distributions when IEI is based on the AIDS model. These results are given in Figures 3 and 4 for the homogeneity and symmetry imposed and relaxed AIDS model respectively. One thing which emerges from this latter comparison is that the distributions look much similar, whether or not the restrictions are imposed. This is reflected by the information provided by the AKS indices in Tables 4 and 5, which indicates that there is not a *statistically significant* difference in the AKS indices from 1978 to 1986, according to either the Table 4 or Table 5 data. However, the distribution of IEI under AIDS model estimation is bimodal, unlike the unimodal distribution for the JLS model. Of course, this is a feature which one would never be aware of if one relied only on the AKS indices. This is one of the reasons why it is useful to consider other measures of inequality, such as comparisons of IEI deciles, which will be discussed below.

Before turning to the decile comparisons, however, a number of further comparisons of AKS indices are of interest, as are the corresponding comparisons of the associated distributions which go along with the AKS indices themselves. What would be the pattern of inequality which would emerge, for example, if one focused on household income rather than IEI? Table 6 provides a comparison of AKS indices across the years 1978–1986, based on total household expenditures ("income") for the goods included in the demand system on which the IEI estimates are based. From this, it can be seen that inequality again does not seem to be changing significantly over time, but also that absolute inequality seems to be lower (as indicated by AKS indices closer to zero, when compared to corresponding values for the JLS or AIDS based AKS index estimates in Tables 2–5). The absence of a statistically significant change in inequality over time as indicated by the AKS indices in Table 6 is confirmed when looking at the income distribution for 1978 and 1986. This is presented in Figure 5, where one can see that there is little change in the proportion of households in different ranges of the distribution from 1978–1986.

The comparisons in Table 6 and Figure 5, based on actual incomes in the various years, is not strictly valid. The households in each of the yearly sub-samples are not all of the same size, and there has been no adjustment for these household size differences. However, the relative pattern of inequality over time is basically the same when focusing on Table 2–5 or Table 6 data. The only difference is that absolute inequality appears lower when basing the AKS index estimates on actual income. An obvious way to control for the problem that there are households of different sizes in the various samples is to use the sub-samples of different household sizes in each year to compute alternative estimates of inequality based on the AKS indices (and later, on the decile ordinate estimates).

Table 7 presents the AKS indices based on actual income for family size F2 (married couples with no children). Here, inequality again appears to rise over time, although not statistically significantly. Also, the level of absolute inequality is lower here than for the entire sample, including all family sizes, reported in Table 6. AKS indices were also calculated for the other family sizes, based on actual income. These estimates were qualitatively similar to those in Table 7, indicating increasing inequality from 1978–1986, except for family size F1 (unattached individuals) and F7 (married couples with five or more children). Again, however, these increases (and decreases) in inequality were not statistically significantly different, given the magnitudes of the AKS index standard errors. Figure 6 provides the graphs of the income distributions for the F2 households, based on their actual incomes, in 1978 and 1986. As the AKS index data in Table 7 indicate, there was not a major change in the distribution of income (in terms of inequality in the distribution of income) between these two years. The IEI-based estimates of inequality based on AKS indices in Tables 2–5 therefore show higher absolute inequality, relative to the estimates based on actual income, even when conditioned on household size. This suggests that care should be used when relying on estimates of inequality based on actual income, even if such estimates are controlled for by household size.

The fact that the pattern of absolute inequality is different when based on actual income versus IEI is not surprising. However, one might expect that the pattern of inequality, when based on IEI for families of different sizes would be fairly close to what one observes with measures based on families of all sizes (as in Tables 2–5). This is not guaranteed, however, since there are other household characteristics which play a role in determining the values of IEI, as mentioned earlier.

It will be recalled that inequality appeared to rise from 1978–1986, when based on JLS and AIDS based estimates of IEI for all households together. (This rise was not statistically significant, however). When considering the AKS indices for family sizes F1–F7, however, this pattern no longer holds, with the exception of the JLS model based estimates, with homogeneity and symmetry relaxed. All other estimates indicate a *declining* pattern of inequality from 1978–1986, for family sizes F3–F6. Family size F2 is the only one where a common pattern of rising inequality is seen across all four estimates of IEI[6]. This is, of course, what was observed for F2 based on actual income. In all cases, the absolute values of the indices are higher for the F2 only samples, compared to the corresponding indices for the full samples, reported in Tables 2–5. Also, the indices are lower when based on homogeneity and symmetry imposed estimated demand models.

The results for the F2 households can be contrasted with those for F3 households which exhibit a *declining* pattern of inequality for all estimates except those based on the JLS model with homogeneity and symmetry relaxed. Not only do these estimates indicate a declining pattern of inequality from 1978–1986, but also the absolute values of the indices are lower than corresponding values in Tables 2–5, for the complete sample. The AKS indices are again lower when based on the homogeneity and symmetry estimated demand models. The densities for the JLS-based cases are similar to those for the complete sample, and for the F2 family size households; but the AIDS-based graphs are distinctly different from the corresponding Figures 3 and 4 (for the complete sample). That is, the distributions are unimodal for F3. However, although inequality is declining, this does not seem to be statistically significant, based on the AKS indices and their standard errors[7].

There are several general conclusions which emerge from the various comparisons in this sub-

[6]Tables of estimated AKS indices for F2 JLS and AIDS models with and without homogeneity and symmetry imposed are available on request. Figures with nonparametrically-estimated plots of the corresponding IEI densities are also available. These plots are similar in appearance to those for the full sample of households, in Figures 1–4.

[7]Again, tables of these indices are available from the author on request. Figures with corresponding plots of the estimated IEI densities for this (F3) household group are also available.

section. First of all, when AKS indices are based on IEI estimates, the pattern on inequality is lower the more restrictive the model estimated. This is true when looking at estimates based on all households, or on the F2 and F3 family sizes. Inequality appears to be increasing from 1978–1986 when based on the complete samples, although this is not statistically significant, if one takes into account the standard errors associated with the AKS indices. This pattern is borne out when computing "naive" AKS indices, based on actual income.

Using actual income to calculate AKS indices is not strictly correct, owing to differences in households' characteristics. AKS indices computed for individual family sizes, however, also indicate increasing inequality from 1978–1986 (although this is still not a statistically significant rise). While calculating AKS indices based on actual income is more appropriate when conditioned on household size, such an adjustment ought not to be necessary for AKS indices based on IEI. A comparison of AKS indices based on IEI controlled for by family size, however, indicates that inequality does not have the same pattern for households of the same size, compared to AKS indices based on all households. For many family sizes, AKS indices based on IEI indicate *declining* inequality from 1978–1986. Thus, when accounting for the most important household characteristic (family size), and re-calculating inequality indices, a different pattern of inequality appears to be present than when using all households' IEI estimates. It should be noted, however, that there are other household characteristics which play a role in estimation of IEI, in addition to family size. These results could thus indicate that basing estimates of inequality on IEI conditioned on family size gives a misleading picture of inequality, since one should control for the other household characteristics too, if conditioning on these after calculating IEI.

Some graphs of IEI and actual income distributions were also presented. These provided a broader perspective on the distribution of income (whether IEI or actual) than the AKS indices, which only provide a summary measure. It was clear from these graphs that the concentration of income was similar whether using the AKS indices or the graphs. However, it was also clear that some interesting features of the distributions were obscured by looking only at the AKS indices. In particular, several graphs indicated multimodal distributions. This could be the nature of the distributions, or reflect that the distributions being considered are mixtures of more than one unimodal distribution. In any event, these contrasts indicate that it is of interest to compare the distributions at a more detailed level, such as the decile estimates which will be considered in the next sub-section.

*Section III.3: IEI and Lorenz Curve Inference*

The summary measures of inequality in IEI and actual income in the preceding sub-section do not give detailed information on the nature of inequality. Varying $r$ and observing what happens to the AKS index gives the impression that inequality is concentrated amongst low income households. Estimates of Lorenz curve ordinates can give a better understanding of this.

IEI decile ordinate vectors were calculated using estimated IEI from each of the four models: the JLS and AIDS models with and without homogeneity and symmetry imposed, and in each of the four years, 1978, 1982, 1984 and 1986. Using the estimated variance-covariance matrices of these vectors, across-year comparisons can be made. Inferential statements can then be made regarding whether observed differences are statistically significant. The results of these comparisons are in Table 8. All comparisons indicate that the Lorenz curves are statistically significantly different at any reasonable significance level, except when comparing those for 1982 with 1978.

Tables 9 and 10 present analogous results to those in Table 8, but for IEI decile ordinates based on F2 and F3 household sizes respectively. This comparison is provided since the F2 and F3 household sizes were the focus of additional attention in the preceding sub-section. From these tables it can be seen that there is really no statistically significant change in inequality from 1978–1986, for either F2 or F3. However, there are statistically significant changes from 1978–1982; and from 1982–1984 for F2 (but not F3). These results again highlight the possibility of conflict when not all household characteristics are taken into account when formulating estimates of inequality based on IEI.

The results in Table 8 formally confirm the earlier results using the AKS indices: that the distribution of IEI is changing over time. To determine *how* the distribution is changing, it is necessary to compare IEI income shares at specific points in the distribution. Results of this kind are reported in Table 11 for the JLS model with and without homogeneity and symmetry imposed respectively.

If “low income households”, “mid-income households” and “high income households” are denoted as those with IEI in deciles 0.10–0.30, 0.40–0.70 and 0.80–1.00 respectively, Table 11 shows that IEI shares are mostly declining for low income households from 1978–1986. This is also true of mid-income households. These patterns are virtually the same whether looking at estimates based on the restricted or unrestricted JLS model. The high income households enjoy a growing share of income. To summarise Table 11, households with the lowest 90% of IEI hold 60% of IEI conditional on the restricted model, but only 10% of IEI conditional on the unrestricted model.

Table 12 contains comparisons of IEI shares based on IEI estimates from the unrestricted and restricted AIDS model. Here again, low and middle income households show a mostly declining share of IEI from 1978–1986, whether the IEI estimates are based on the restricted or unrestricted AIDS model. High income households exhibit a growing share of IEI. For estimates based on either the restricted or unrestricted model, however, households with the lowest 90% IEI hold between 70–74% of IEI, irrespective of year.

Corresponding results to those in Table 12 were also calculated for F2 and F3 (these are available on request). These results confirm the Table 9 and 10 results: that inequality increases from 1978–1982; and from 1982–1984 for F2, but that there is no change over the whole period for F3 households. The largest differences are concentrated in the fourth to eighth deciles for F2, which show a decrease then an increase in inequality from 1978–1982; then from 1982–1984.

These results are also qualitatively similar to the results for the AKS indices. That is, estimates are much closer for the restricted versus unrestricted AIDS model, relative to the same comparison for the JLS model. Thus, although the AIDS model is much more restrictive than the JLS model, the results based on the former are more robust to changes in model specification than the latter.

These more detailed results thus confirm what was suggested by the pattern of the AKS indices in Tables 2–5. That is, inequality in IEI is rising over time. In addition, this inequality seems to be concentrated in the lowest 70% of the IEI distribution. Although *absolute* inequality seems to be greatest when looking at estimates of IEI based on the least restrictive model, the pattern of *relative* inequality is robust to major differences in model specification and to inequality indices based on actual income. The approach used in this paper thus appears to capture relative inequality in the distribution of welfare, represented by inequality in the distribution of IEI. However, if one is interested in absolute levels of inequality, then some improved model specification to calculate IEI is required. This can be difficult, since IEI must be calculated using a demand model with IB/ESE GES. Many commonly used demand models do not incorporate these restrictions. However, a rank three demand model with IB/ESE GES is an obvious candidate for analysis. This is the subject of ongoing research.

## Section IV: Summary and Conclusions

Inequality is difficult to measure for many reasons. For example, household income alone can give a misleading picture of inequality since households differ in their characteristics. Also, the inequality measure chosen will embody a normative judgement regarding the "weights" which different individuals are assigned.

In this paper, measures of inequality in the distribution of household welfare are calculated which circumvent some of the usual difficulties. Using a demand model with IB/ESE GES, IEI can be estimated. IEI adjusts household income to account for differences in households' characteristics. Also, by definition, IB/ESE GES do not depend on utility, since they embody restrictions which uniquely identify the GES. That is, a specific set of restrictions is imposed which selects one out of the infinite set of GES which could be estimated. The IEI so calculated are conditional on a specific level of interpersonal comparison, so they can be interpreted as indicators of household welfare.

The demand models used to calculate IEI were the JLS and AIDS, both of which included IB/ESE GES. Each model was estimated with and without homogeneity and symmetry restrictions imposed. This enables a determination of the sensitivity of inequality in IEI to differences in model specification. This is important because there is a large variety of demand models one can choose from, but only a relatively more limited number which can be used to estimate IEI uniquely from IB/ESE GES.

All estimation was based on data from the 1978, 1982, 1984 and 1986 *Survey of Family Expenditure Survey Microdata Files* for Canada. This meant that inequality could be measured for each of these years, based on IEI calculated using parameter estimates from the four demand model parameterisations.

Two kinds of inequality measure were computed. A series of summary index measures of the AKS type, and more detailed estimates of inequality based on estimated IEI decile ordinate vectors and their variance-covariance matrices. The IEI distributions were also graphed, by estimating the distributions using a nonparametric kernel method.

It was found that inequality increased over time based on the AKS indices of inequality, no matter what the value of the inequality aversion parameter. Inequality also rose over time no matter which specification of the demand model was used. Thus, relative inequality in IEI increased significantly from 1978 to 1986. However, the least restrictive model (the JLS model with homogeneity and symmetry relaxed) suggested a greater degree of absolute inequality than the most restrictive model (the AIDS model with homogeneity and symmetry imposed). Inequality based on estimates of Lorenz curve ordinates of IEI indicated that the distribution of IEI changed statistically significantly from 1978 to 1986. In addition, it was seen that inequality grew for households in the bottom 70% of the IEI distribution. The absolute extent of this inequality appeared *greater* the *less restrictive* the demand model used to estimate IEI. This supports the earlier result using the AKS indices.

The effect of calculating inequality estimates after appropriately adjusting for differences in household characteristics was also contrasted with estimates based on actual income. While relative inequality was similar on this basis, absolute inequality was much lower (a more even distribution) when based on actual income. This indicates that "naive" estimates of inequality which do not adjust for differences in household characteristics yield estimates of inequality which are much more egalitarian than is truly the case. This finding also held when basing estimates of inequality on actual income by households of the same size. When calculating AKS indices of inequality on sub-samples of households of the same size using IEI, however, a different pattern of inequality seemed to be present. For households bigger than F3, inequality seemed to be *declining* over time. However, such estimates are likely misleading, since IEI controls for more than simply family size.

These results indicate that estimation of *relative* inequality in IEI is very robust to demand model specification, but estimates of *absolute* inequality are not. This is a useful result, as policy-makers are often more interested in the relative level of inequality, in trying to ensure "fairness" in the tax system, for example. This is not to say that absolute inequality estimation is not important, but rather that it is somewhat more difficult to measure accurately. This could be achieved by working with demand models which continue to include IB/ESE GES, yet do not exhibit model specification error, such as rejection of homogeneity and symmetry. This is the subject of ongoing research.

**References**


Atkinson, A. (1970), "On the Measurement of Inequality." *Jounral of Economic Theory*, **2**, 244–263.

Banks, J., R. Blundell and A. Lewbel (1994), "Quadratic Engel Curves, Indirect Tax Reform and Welfare Measurement." University College London, Department of Economics Working Paper No. 94–04.

Barnes, R. and R. Gillingham (1984), "Demogrpahic Effects in Demand Analysis: Estimation of the Quadratic Expenditure System." *Review of Economics and Statistics*, **66**, 591–601.

Beach, C.M and R. Davidson (1983), "Distribution-Free Statistical Inference With Lorenz Curve and Income Shares." *Review of Economic Studies*, **50**, 723–735.

Blackorby, C. and D. Donaldson (1982), "Ratio-Scale and Translation-Scale Full Interpersonal Comparability Without Domain Restrictions: Admissible Social Evaluation Functions." *International Economic Review*, **23**, 249–268.

Blackorby, C. and D. Donaldson (1993a), "Adult-Equivalence Scales and the Economic Implementation of Interpersonal Comparisons of Well-Being." *Social Choice and Welfare*, **10**, 335–361.

Blackorby, C. and D. Donaldson (1993b), "Household Equivalence Scales and Welfare Comparisons: A Comment." *Journal of Public Economics*, **50**, 143–146.

Blundell, R., P. Pashardes and G. Weber (1993), "What Do We Learn About Consumer Demand Patterns From Micro Data." *American Economic Review*, **83**, 570–597.

Bollino, C.A, and R. Violi (1990), "GAITL: A Generalised Version of the Almost Ideal Demand System." *Economics Letters*, **33**, 127–129.

Davies, J. and M. Hoy (1995), "Making Inequality Comparisons When Lorenz Curves Intersect." *American Economic Review*, **85**, 980–986.

Deaton, A. and J. Muellbauer (1980), "An Almost Ideal Demand System." *American Economic Review*, **70**, 312–326.

Härdle, W. (1990), *Applied Nonparametric Regression.* Cambridge University Press.

Jorgenson, D. W., L. J. Lau and T. M. Stoker (1982), "The Transcendental Logarithmic Model of Aggregate Consumer Behavior." In *Advances in Econometrics* (Vol. 1), R. L. Basmann and G. F. Rhodes, eds., JAI Press, Greenwich, CT, pp. 97–238.

Jorgenson, D. W. and D. T. Slesnick (1984), "Aggregate Consumer Behavior and the Measurement of Inequality." *Review of Economic Studies*, **51**, 369–392.

Jorgenson, D. W. and D. T. Slesnick (1987), "Aggregate Consumer Behavior and Household Equivalence Scales." *Journal of Business and Economic Statistics*, **5**, 219–232.

Keuzenkamp, H.A. and A.P. Barten (1991), "Rejection Without Falsification: On the History of Testing the Homogeneity Condition in the Theory of Consumer Demand." Unpublished paper, presented at the European Meetings of the Econometric Society, Cambridge, England.

Kolm, S. (1976a), "Unequal Inequalities I," *Journal of Economic Theory*, **12**, 416–442.

Kolm, S. (1976b), "Unequal Inequalities II," *Journal of Economic Theory*, **13**, 82–111.

Kwong, K-S. (1987), "The Distributive Impact of Price Changes in Canada." Unpublished mimeo.

Lewbel, A. (1985), "A Unified Approach to Incorporating Demographic Effects into Demand Systems." *Review of Economics Studies*, bf 52, 1–18.

Lewbel, A. (1989a), "Household Equivalence Scales and Welfare Comparisons." *Journal of Public Economics*, **39**, 377–391.

Lewbel, A. (1989b), "Nesting the AIDS and Translog Demand Systems." *International Economic Review*, **30**, 349–356.

Lewbel, A. (1991), "The Rank of Demand Systems: Theory and Nonparametric Estimation." *Econometrica*, **59**, 711–730.

Lewbel, A. (1993), "Household Equivalence Scales and Welfare Comparisons: Reply." *Journal of Public Economics*, **50**, 147–148.

Nicol, C.J. (1989a), "Testing a Theory of Exact Aggregation." *Journal of Business and Economic Statistics*, **7**, 259–265.

Nicol, C.J. (1989b), "A Reinterpretation of the Almost Ideal Demand System." In *Advances in Econometrics and Modelling*, B. Raj, ed., pp. 117–129. Kluwer Academic Publishers, Amsterdam.

Nicol, C.J. (1991), "Aggregate Consumer Behaviour Without Exact Aggregation." *Canadian Journal of Economics*, **24**, 578–594.

Nicol, C.J. (1995a) "Model Specification and Estimation Effects in Applied Demand Analysis Using Microdata". Department of Economics Working Paper No. 57, University of Regina.

Nicol, C.J. (1995b) "Model Specification Issues in Consumer Demand Systems Using United States Microdata". Department of Economics Working Paper No. 56, University of Regina.

Nicol, C.J. (1996), " A Comparison of Demand Specifications Estimated Using Canadian and United States Microdata." Unpublished mimeo. University of Regina.

Nicol, C.J. (1994) "Identifiability of Household Equivalence Scales Through Exact Aggregation: Some Empirical Results." *Canadian Journal of Economics*, **27**, 307–328.

Ramezani, A., D. Rose and S. Murphy (1995) "Aggregation, Flexible Forms and Estimation of Food Consumption Parameters." *American Journal of Agricultural Economics*, **77**, 525–532.

Sen, A.K. (1973), *On Economic Inequalty*, Clarendon Press, Oxford.

Survey of Family Expenditures Microdata Files (1978, 1982 1984 and 1986). Family Expenditure Surveys Section, Statistics Canada, Ottawa, Canada.

Thistle, P.D. (1990), "Large Sample Properties of Two Inequality Indices." *Econometrica*, **58**, 725–728.

Vuong, Q.H., (1989), "Likelihood Ratio Tests for Model Selection and Non-nested Hypotheses." *Econometrica*, **57**, 307–333.

Wang, Q., C. Halbrendt and S.R. Johnson (1996), "A Non-nested Test of the AIDS vs. the Translog Demand System." *Economics Letters*, **51**, 139–143.