CURRY, Haskell Brooks (1900–1982)

Haskell Brooks Curry was born on 12 September 1900 at Millis, Massachusetts and died on 1 September 1982 at State College, Pennsylvania. His parents were Samuel Silas Curry and Anna Baright Curry, founders of the School for Expression in Boston, now known as Curry College. He graduated from high school in 1916 and entered Harvard College with the intention of going into medicine. When the USA entered World War I in 1917, he changed his major to mathematics because he thought it might be useful for military purposes in the artillery. He also joined the Student Army Training Corps on 18 October 1918, but the war ended before he saw action. He received his B.A. in mathematics from Harvard University in 1920. From 1920 until 1922, he studied electrical engineering at MIT in a program that involved working half-time at the General Electric Company. From 1922 until 1924, he studied physics at Harvard; during the first of those two years he was a half-time research assistant to P. W. Bridgman, and in 1924 he received an A. M. in physics. He then studied mathematics, still at Harvard, until 1927; during the first semester of 1926–27 he was a half-time instructor. During 1927–28 he was an instructor at Princeton University. During 1928–29, he studied at the University of Göttingen, where he wrote his doctoral dissertation; his oral defense under David Hilbert was on 24 July 1929, although the published version carries the date of 1930.

In the fall of 1929, Haskell Curry joined the faculty of The Pennsylvania State College (now The Pennsylvania State University), where he spent most of the rest of his career. However, there were some exceptions: 1) In 1931–32, he was at the University of Chicago as a National Research Council Fellow; 2) In 1938–39, he was in residence at the Institute for Advanced Study at Princeton; 3) From 1942–46, he worked for the United States government doing applied mathematics for the war effort, first at the Frankford Arsenal (1942–1944), then at the Applied Physics Laboratory at John Hopkins University (1944–1945), and finally at the Ballistic Research Laboratories at the Aberdeen Proving Ground (1945–1946), where he worked on the ENIAC project; 4) In 1950–51 he was at the University of Louvain in Belgium on a Fulbright grant. In 1960, he became Evan Pugh Research Professor of Mathematics at the Pennsylvania State University, and this enabled him to devote most of his time to his research. It also enabled him to take a trip around the world in 1962, visiting a number of universities and giving talks. He retired from The Pennsylvania State University in 1966 and went to the University of Amsterdam in the Netherlands, where he stayed until 1970. He then returned to The Pennsylvania State University, where, except for a visit to the University of Pittsburgh in 1971–72, he remained until his death.

Haskell Curry's main work was in the field of mathematical logic. On 20 May 1922, when he was 21 years old, he looked at the first chapter of Whitehead and Russell's *Principia Mathematica*, and he noticed that the system was based on two rules: modus ponens and substitution of well-formed formulas for propositional variables. He also noticed immediately that this rule of substitution was significantly more complicated than modus ponens, more complicated to describe and use, and also more complicated in the sense of computer implementation (although this was long before there were either computers or computer implementations of logical systems). He wanted to break the rule of substitution down into simpler rules, and his work on this led him to what he called "combinatory logic," which became the main part of his life's work. Since he was working on a new foundation for logic, he had to begin with a discussion of the nature of these foundations, and his early ideas on this appear in his first two papers: "An Analysis of Logic Substitution" (1929) and "Grundlagen der Kombinatorischen Logik" (1930). Here Curry first defines what he means by a formal system (which he originally called an "abstract theory"). Stating clearly that such a theory does not involve meaningless symbols, he indicates that such a theory is based on a "primitive frame," which is to specify the formal objects considered by the theory, what it means to say that a formal object is asserted, and which assertions are true. Thus, for Curry, what was proved in an abstract theory were not formal objects, but statements, which were of the form "*X* is asserted" or "|– *X*," where *X* is a formal object; these statements are formed from the formal objects by the predicate "is asserted." The formal objects and the true assertions are inductively defined sets, and although he does not emphasize the point at this stage, the formal objects need not be words on an alphabet. In "Grundlagen der Kombinatorischen Logik," he makes the point that he wants to have in his system formal objects which represent the paradoxes, for example, if F is the property of properties φ for which $F(\varphi)$ = not $\varphi(\varphi)$, then $F(F)$ = not $F(F)$, and so $F(F)$ represents the liar paradox. Curry proposes here to avoid the paradox by denying that $F(F)$ is a proposition, but he wants this fact about $F(F)$ not being a proposition to be a theorem of his logic and not to have the formal object involved excluded from the theory by the rules of formation.

Following "Grundlagen der Kombinatorischen Logik," Curry published a series of technical papers continuing the development of combinatory logic. In one of them, "First Properties of Functionality in Combinatory Logic" (1936, but written in 1932), he introduced into combinatory logic machinery for treating grammatical or logical categories, such as "proposition," and showed how certain paradoxes could be avoided by the postulates adopted for these formal objects. In a footnote in this paper, he stated the view that a proof of absolute consistency is a secondary matter for the acceptability of a theory. He concluded that a theory of logic should be judged as a whole, and should be treated as a hypothesis, which can be accepted as long as it remains useful. Clearly the discovery of a contradiction would make a theory useless, but in the absence of a contradiction, a proof of consistency is not really needed.

In 1932–33, Alonzo Church published "A Set of Postulates for the Foundation of Logic," which contained a system with a basis very similar to Curry's combinatory logic. Then, Church's students, Kleene and Rosser, in "The Inconsistency of Certain Formal Logics" (1935), proved the inconsistency of Church's original system and the extension of Curry's original system that appeared in "Some Properties of Equality and Implication in Combinatory Logic" (1934). Church and his students responded by abandoning the idea of basing logic and mathematics on this kind of system, and they abstracted from Church's system the λ-calculus, which is equivalent to basic combinatory logic (without any logical connectives and quantifiers). For Curry, however, who had already considered the possibility of a contradiction developing as he extended his system, the contradiction discovered by Kleene and Rosser only meant that a contradiction could be derived from weaker assumptions than he had realized; his ideas about the prelogic and the category of propositions gave him a means of searching for a system that would be consistent, but now he felt he needed a consistency proof. As part of this work, he discovered, in "The Inconsistency of Certain Formal Logics" (1942), a much simpler contradiction than that of Kleene and Rosser which would follow from the same assumptions as theirs.

Meanwhile, in response to a request to present a paper on the subject to a meeting of the International Congress for the Unity of Science, which was held at Cambridge, Massachusetts at the beginning of September, 1939, he began to write independently on his philosophical ideas. His first manuscript was too long for the meeting, and was eventually published with only minor revisions in 1951 as *Outlines of a Formalist Philosophy of Mathematics.* But the manuscript served as the basis for the paper he did present at the meeting, "Remarks on the Definition and Nature of Mathematics" (1939). In these works, Curry proposed that mathematics be defined as the science of formal systems. He intended this to be an alternative to (naive) realism, which holds that mathematics is about objects that exist in the physical world, and idealism, which holds that mathematics is about mental objects. His examples of idealism in mathematics were Platonism and intuitionism. In order to explain this definition of mathematics, Curry devoted a considerable amount of space, especially in *Outlines of a Formalist Philosophy of Mathematics*, to an exposition of his idea of formal systems, with many examples. This was essentially the definition he had given of an abstract theory in "An Analysis of Logical Substitution" and "Grundlagen der

Kombinatorischen Logik," except that in addition to the one-place predicate assertion he allowed for other predicates with other arities, so that the statements of a formal system could, for example, be equations. In addition to this definition, Curry pointed out that a study of formal systems is not limited to deriving one formal theorem after another, but also involves proving metatheorems, or general theorems about the system. In *Outlines of a Formalist Philosophy of Mathematics*, he suggests that this metatheory itself might be formalized. Indeed, he had already started a project of justifying the rules of standard logical systems in natural deduction formulations by taking the systems as formalizing the elementary metatheory of an elementary formal system. He did this first in an unfinished manuscript "Some Properties of Formal Deducibility" (1937). This project was eventually finished and published as *A Theory of Formal Deducibility* (1950), where Gentzen-style L-formulations were also considered, and this was all expanded in *Foundations of Mathematical Logic* (1963).

Curry also made a point of distinguishing between truth within a system or theory, which depends on the definition of the system or theory, and the acceptability of the system or theory for some purpose, which depends on the purpose. Curry called this philosophy of mathematics "formalism."

Curry elaborated these views in the paper "Some Aspects of the Problem of Mathematical Rigor" (1941). In this paper, he emphasized the distinction between the formal objects and their representations; these representations may be strings of letters, but they may also be entirely different things. In addition, he contrasted a formal system to a calculus, which is a class of symbols of a language, the object language, together with rules for their manipulation. Curry indicated how to convert mechanically a formal system into a calculus and a calculus into a formal system. Finally, he coined the word "contensive" to refer to realist or idealist philosophies of mathematics (intending it as a translation for the German word "inhaltlich"). In his view, mathematics is not contensive, that is, it does not have any essential subject matter, and the only subject matter of mathematical propositions is other mathematics.

In a series of further papers, he elaborated these ideas and related them to the ideas of others, including Carnap and Lorenzen. The ideas were further elaborated into the introductions to his books *A Theory of Formal Deducibility*, *Leçons de Logique Algébrique* (1952), *Combinatory Logic*, vol. I (with Robert Feys, 1958), *Foundations of Mathematical Logic*, and *Combinatory Logic*, vol. II (with J. R. Hindley and J. P. Seldin, 1972). As he did this, he refined his notion of formal system so that it would include some versions of calculi. As part of this process, he revised his terminology as others criticized his use of some terms. Thus, for example, his original term for formal object in the 1920s and early 1930s was “entity” in English and “Etwas” (used as a noun) in German. When a philosopher told him that his use of the English word “entity” involved some philosophical assumptions that he did not with to make, he stopped using it (personal oral communication), and in *Outlines of a Formalist Philosophy of Mathematics* used the English word “term.” However, his use of the word “term” for the formal object was in conflict with the use of that word in systems of predicate logic, and so starting about 1950 he substituted for it a word of his own: “ob.” (See the preface to the second edition of *A Theory of Formal Deducibility*, 1957.) But not all formal objects are obs in this sense; obs are formal objects with the property that each one has a unique construction from the atomic formal objects. In a system in which the formal objects are words on an alphabet, not every word has a unique construction, since the word “abc” can be constructed by adding “c” to the right of “ab” or by adding “a” to the left of “bc.” Curry thus distinguished two kinds of formal systems: *ob systems*, in which every formal object has a unique construction from the atomic formal objects, and *syntactical systems*, in which the formal objects are words on an alphabet. The standard systems of propositional and predicate logic are both, since the formal objects are, indeed, words on an alphabet, but in all the standard definitions, the “well-formed formulas” all have a unique construction from the atomic formulas.

Another example is an objection raised by Kleene to Curry’s use of the prefix “meta-,” which Kleene felt should apply only to symbols and languages. So, starting about 1951, Curry began systematically using the prefix “epi-“ instead of “meta-.” (See the preface of *Outlines of a Formalist Philosophy of Mathematics*.) This idiosyncratic use of words has made some of Curry’s works difficult to read, and has caused some misunderstanding of some of his ideas. (For more on Curry’s notion of formal system, see J. P. Seldin, “Arithmetic as a Study of Formal Systems,” 1975. For more on the history of Curry’s terminology, see *Foundations of Mathematical Logic*, pp. 85–86.)

This process involved other changes in Curry’s views. By the early 1960s, Curry was no longer saying that mathematics is the science of formal *systems*, but that it is the science of formal *methods* (see *Foundations of Mathematical Logic*, p. 14). This revised definition can be used to answer a criticism made on several occasions that under Curry’s (earlier) definition there could have been no mathematics before there were formal systems. In “The Purposes of Logical Formalization” (1968), Curry compares the process that led from the nineteenth century arithmetization of analysis to twentieth century formal logic to the process that led from the informal seventeenth and eighteenth century deductions in calculus to the more formal ε–δ proofs of 19th century analysis. This indicates that Curry considered the introduction of ε–δ proofs to be a kind of formalization, and that therefore, for Curry, formal methods go far beyond formal systems.

Curry believed strongly that mathematics is like language, a creation of human beings. He also thought that what was thus created had objective existence after it was created. Thus, for Curry, mathematics belonged to what Karl Popper called the "third world." In fact, when Popper presented his ideas on this in his paper "Epistemology Without a Knowing Subject" at the Third International Congress for Logic, Methodology and Philosophy of Science in 1967, it was at a session which Curry chaired. Curry's immediate reaction to Popper's paper was that it made a big deal out of something that was trivially true. (Personal oral communication at the time.)

Curry felt the need to use the word "formalism" for his philosophy of mathematics, probably because he studied at Göttingen under Hilbert, but his philosophy actually appears to be a form of structuralism (see S. Shapiro, *Thinking About Mathematics*, 2000, Chapter 10).

BIBLIOGRAPHY

"An Analysis of Logical Substitution," *American Journal of Mathematics* 51 (1929): 363–384.

"Grundlagen der Kombinatorischen Logik (Inauguraldissertation)," *American Journal of Mathematics* 52 (1930): 509–536, 789–834.

"Some Properties of Equality and Implication in Combinatory Logic," *Annals of Mathematics* (2) 35 (1934): 849–860.

"First Properties of Functionality in Combinatory Logic," *Tôhoku Mathematical Journal* 41 Part II (1936): 371–401.

"Some Properties of Formal Deducibility (Abstract)," *Bulletin of the American Mathematical Society* 43 (1937): 615.

"Remarks on the Definition and Nature of Mathematics," *Journal of Unified Science* 9 (1939): 164–169. Reprinted *Dialectica* 8 (1954): 228–233. Reprinted again in Paul Benacerraf and Hilary Putnam, editors, *Philosophy of Mathematics: Selected Readings*, (Englewood Cliffs, NJ, 1964). pp. 152–156.

"Some Aspects of the Problem of Mathematical Rigor," *Bulletin of the American Mathematical Society* 47 (1941): 221–241.

"The Inconsistency of Certain Formal Logics," *Journal of Symbolic Logic* 7 (1942): 115–117.

A Theory of Formal Deducibility (Notre Dame, IN, 1950; Second Edition 1957).

Outlines of a Formalist Philosophy of Mathematics (Amsterdam, 1951).

Leçons de Logique Algébrique (Paris and Louvain, 1952).

(With R. Feys) *Combinatory Logic*, vol. I (Amsterdam, 1958, reprinted 1968, 1974).

Foundations of Mathematical Logic (New York, 1963; reprinted 1977).

(With J. R. Hindley and J. P. Seldin) *Combinatory Logic*, vol II (Amsterdam, 1972).

"The Purposes of Logical Formalization," *Logique et Analyse* 11 (1968): 357–366.

Other Relevant Works

"Language, Metalanguage, and Formal System," *Philosophical Review* 59 (1950): 346–353.

"L-semantics as a Formal System," in *Congrès International de Philosophie des Sciences, Paris 1949, II Logique, Actualités Scientifique et Industrielles 1134* (Paris, 1951), pp. 19–29.

"Philosophische Bemerkungen zu Einigen Problemen der Mathematischen Logik," *Archiv für Philosophie* 4 (1951): 147–156.

"On the Definition of Substitution, Replacement, and Allied Notions in an Abstract Formal System," *Revue Philosophique de Louvain* 50 (1952): 251–269.

"Mathematics, Syntactics, and Logic," *Mind* ns 62 (1953): 172–183.

"Theory and Experience," *Dialectica (Neuchatel)* 7 (1953): 176–178.

"Calculuses and Formal Systems," *Dialectica* 12 (1958): 249–273.

"Logic as a Mathematical Science," *Annals of the Japan Association for the Philosophy of Science* 2 (1963): 131–143.

"The Relation of Logic to Science," in S. Dockx and P. Bernays, editors, *Information and Prediction in Science* (New York, 1965), pp. 79–96.

"The Equivalence of Two Definitions of Elementary Formal System," *Compositio Mathematicae* 20 (1968): 13–20.

"Some Philosophical Aspects of Combinatory Logic," in J. Barwise, H. Jerome Keisler, and K. Kunen, editors, *The Kleene Symposium: A Collection of Papers on Recursion Theory and Intuitionism* (Amsterdam, 1980), pp. 85–101.

Further Reading

A. Church, "A Set of Postulates for the Foundation of Logic," *Annals of Mathematics* (2) 33 (1932): 346–366, Second paper, *Ibid.* 34 (1933): 839–864.

S. C. Kleene and J. B. Rosser, "The Inconsistency of Certain Formal Logics," *Annals of Mathematics* (2) 36 (1935): 630–636.

K. R. Popper, "Epistemology Without a Knowing Subject," in B. van Rootselaar and J. F. Staal, editors, *Proceedings of the Third International Congress for Logic, Methodology and Philosophy of Science* (Amsterdam, 1968), pp. 333–373. Reprinted in K. R. Popper, *Objective Knowledge* (Oxford, 1972), pp. 106–152.

J. P. Seldin, "Arithmetic as a Study of Formal Systems," *Notre Dame Journal of Formal Logic* 16 (1975): 449–64.

J. P. Seldin, "Curry's Program," in J. P. Seldin and J. R. Hindley, editors, *To H. B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism* (London, New York, etc., 1980), pp. 3–33.

S. Shapiro, *Thinking About Mathematics* (Oxford, 2000).

B. Russell and A. N. Whitehead, *Principia Mathematica* (Cambridge, 1910, 1912, 1913).