A RELEVANT VALIDITY IN CURRY'S FOUNDATIONS: A REPLY TO RICHARD SYLVAN

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Sylvan's argument in [3] against Curry's defense of the "positive paradox principle", namely

$$\vdash A \supset (B \supset A),$$

depends on his claim that the principle of $\supset$-introduction,

$$A \vdash B \quad \rightarrow \vdash A \supset B,$$

fails to imply the principle of $\supset$-introduction with parameters,

$$C, A \vdash B \rightarrow C \vdash A \supset B.$$

This argument is based on equating this last principle with deducing

$$\vdash_C A \supset B$$

from the ability to infer $\vdash_C B$ from $\vdash_C A$, where $\vdash_C$ refers to the system obtained from the system to which $\vdash$ refers by adjoining $\vdash C$ as a new axiom. While this argument does take account of what Curry says on pp. 172–3 of [1], it ignores what Curry says elsewhere about his use of the symbol $\vdash$.

On [1] p. 66, Curry says that

$$X_1, \ldots, X_m \vdash Y$$

means that "if $X_1, \ldots, X_m$ are adjoined to the system as new axiomatic obs, then Y is an asserted ob in the extended system." He explains this further in [2] p. 50, where he says that it is an abbreviation for

$$\vdash X_1 \,\&\, \ldots \,\&\, \vdash X_m \Rightarrow \vdash Y,$$

which means that there is a deduction ending in $\vdash Y$ in a system "whose basis is formed by adjoining" $\vdash X_1, \ldots, \vdash X_m$ "to the axioms." From what Curry says on pp. 47ff of [2], this means that there is a tree each of whose nodes is a statement of the form $\vdash T$ where each top node is an axiom or some $\vdash X_i$, where the conclusion is $\vdash Y$, and where each node which is not a top node is obtained from the formulas immediately above it by

one of the rules of the system. (The system is assumed to be elementary, so there is no discharging of assumptions.)

If we apply this to the argument at hand, we see that by

$$C, \quad A \vdash B$$

Curry means that there is a deduction tree each of whose top nodes is an axiom or C or A and whose conclusion is B, where the extra yield symbols have been suppressed. Furthermore, this is exactly what he means by

$$A \vdash_C B$$

(which is *not* the same thing as the ability to infer $\vdash_C B$ from $\vdash_C A$). Thus, what Curry means to assert about the meaning of $\supset$ on pp. 172–173 of [1] is

$$\Gamma, \quad A \vdash B \leftrightarrow \Gamma \vdash A \supset B,$$

where Γ is any set of assumptions.

Thus, in terms of Curry's definitions, the positive paradox principle is valid in terms of the metatheory of elementary formal systems (which is what Curry is talking about on p. 173 of [1]). Curry's claim about the positive paradox principle is thus that it is valid in a particular context in the formal metatheory of elementary formal systems (as he has defined it). As the last paragraph on p. 173 of [1] shows, he is not claiming that it is true generally.

REFERENCES

[1] Curry, H. B., *Foundations of Mathematical Logic,* McGraw-Hill, New York, 1963.

[2] Curry, H. B. & Feys, R., *Combinatory Logic,* vol. I, North-Holland, Amsterdam, 1958.

[3] Sylvan, Richard, A relevant invalidity in Curry's Foundations, BSL 16 (1987), no. 1, 51–53.

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