Informal Incompleteness: Rules, Philosophy, Law*

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Abstract

This paper represents the start of an inquiry into whether Gödel's Incompleteness Theorem and related results have applications outside of the field of mathematical logic. In particular, it raises the question of whether these results imply that there are limits to what theories can be completely and correctly characterized by means of rules. After an introduction to the theorem and an outline of its proof, the paper goes on to raise questions about whether the ideas involved can be applied to philosophy (philosophy of science and ethics) and to the law.

In 1930, the Austrian mathematician and logician Kurt Gödel proved one of the deepest results in mathematical logic, his Incompleteness Theorem. Gödel's original paper is [4]; a modern technical introduction is [12]; a guide in the form of logical puzzles is [11]. Roughly speaking, this theorem states that

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for any strictly formal system that is strong enough to include elementary arithmetic (the elementary theory of the whole numbers), there is a sentence which cannot be proved but which is nevertheless true. This result implies that there is no set of formal rules which we humans can use to completely characterize the true sentences of elementary arithmetic or which any devices that we can build can use for the same purpose. In the years following the publication of Gödel's result, a number of other results appeared that seem to support the idea that there are limits to what can be characterized by strictly formal rules. For example, there is the result that it is not possible to decide whether a computer program run with given input will stop or go into an infinite loop; this result is known as the *undecidability of the halting problem*, originally proved in [13]. (For modern introductions, see [1, Chapter 5, Theorem 2.2] and [2, Chapter 4, Theorem 2.1 and Chapter 6, Theorem 4.1].) One way of looking at these results is that certain programs are too complex to be calculated at all by an idealized computer, even one with unlimited memory and unlimited time. These results are generally taken by mathematical logicians and theoretical computer scientists to show that there are limits to what can be done using formal rules in a strictly formal setting.

In this paper I propose to investigate the possibility that this kind of incompleteness applies to systems of rules that are not completely formalized. The history of the formalization of mathematics is one of examining the arguments used in proofs and making explicit *all* the assumptions on which they depend. The form of many informal deductive arguments seems to imply that they could be formalized by this process, which, in turn, suggests that the theories of which they are a part may also be incomplete in much the sense of Gödel's Incompleteness Theorem. It also suggests that there may be limits on the use of rules to characterize ideas even if the rules are not being used in a completely formal way. My purpose here is to consider the possible effect such incompleteness might have for the use of such theories.

Among the cases I propose to examine are the following:

1. *Philosophical arguments.* Most philosophical arguments are deductive, but not completely formalized. This means that it may be important to consider the possibility that the basic axioms and rules being used in these deductive arguments may not completely characterize the subjects involved. Among the areas of philosophy which may be affected are:

(a) The Philosophy of Science

(b) Ethics

2. *Law.* Law, like philosophy, involves arguments that have a deductive character but are not completely formalized. These arguments also differ in important respects from arguments in such fields as mathematics. My purpose here is to begin an inquiry into whether the ideas of Gödel's Incompleteness Theorem and related results may possibly have applications to the law.

The paper will open with an introduction to Gödel's Incompleteness Theorem itself. To make things easier for the reader, I will start in §1 with an outline of an easier proof, that of the undecidability of the halting problem. To present an outline of the proof of Gödel's Incompleteness Theorem itself, I need to give a complete definition of a system to which it applies: first-order arithmetic. I do this in §2, and then in §3 I will present the outline of the proof of the theorem itself. In §4 I will comment on the the possible application of the theorem to theories which are not strictly formalized. §5 I will consider an application of these ideas to philosophy, and in §6 I will consider applications to law.

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1 The Undecidability of the Halting Problem

Imagine that we have a computer built like existing computers and working on the same principles but which has unlimited time and space (including unlimited memory) at its disposal. It is generally accepted in mathematical logic and theoretical computer science that the functions that can be computed by such a computer are precisely the partial recursive functions.[1]

Now in our idealized computer, as in any existing computer, everything is coded by numbers. Every input or output is stored simply as a number.

[1] This set of functions can also be described as the set of functions computable by a Turing machine, or by the set of all lambda-definable functions. It is not necessary for this paper to understand how any of these classes of functions are defined; it is enough to know that they have all been proved to be the same class of functions, and this is part of the reason that each of these classes of functions is now regarded as the class of functions that can be computed by our idealized computer.

Every program or file can be thought of as a finite sequence of numbers, and there are well-known ways in mathematics for coding finite sequences of numbers as single numbers.

Theorem 1 *There is no program that will determine whether running a given program with a given input of data will terminate or go into an infinite loop.*

Outline of proof[2] Suppose there is such a program, `H(x,y)` so that `H(m,n)` returns `True` if running program `m` with input data `n` terminates and returns `False` otherwise.

Consider the following program:

```
1  Input X
2  If H(X,X) then goto 2
3  Output True
4  End
```

This program runs as follows: it first reads the number `X` from the input. Then, in line 2, it runs the program `H(X,X)`. If `H(X,X)` returns `True`, it is directed to go to line 2, the same line is has just been executing, and so is clearly put into an infinite loop. If, on the other hand, `H(X,X)` returns `False`, the program executes the next line, which means that it outputs `True` and then proceeds to the next line and ends (halts execution). So this program goes into an infinite loop if `H(X,X)` outputs `True` and outputs `False` and halts if `H(X,X)` outputs `False`.

This program is identified with a number, say `N`. Then we clearly have, for every number `X`,

$$\mathtt{H}(\mathtt{N},\mathtt{X}) = \mathtt{True} \Leftrightarrow \mathtt{H}(\mathtt{X},\mathtt{X}) = \mathtt{False}$$

Putting `N` for `X`,

$$\mathtt{H}(\mathtt{N},\mathtt{N}) = \mathtt{True} \Leftrightarrow \mathtt{H}(\mathtt{N},\mathtt{N}) = \mathtt{False}$$

This is a contradiction, since it implies `True = False`.

Because this contradiction follows from the assumption that there is a program `H(x,y)` with the property that `H(m,n)` returns `True` if program `m` with input data `n` halts and returns `False` otherwise, it follows that there is no such program.

[2] This outline is based on [2, Theorem2.1, page 54.]

Remark 1 Note that what this proof really shows is that no matter what rules we write down for the purpose of defining the program `H(x,y)` with the property that `H(m,n)` returns `True` if program `m` with input data `n` halts and returns `False` otherwise, when the program is run on our idealized computer, it simply will not satisfy this specification. Thus, this is not so much a limitation on what programming rules can be written down, but what our idealized computer can accomplish with whatever rules it happens to be given. The key limitation on our idealized computer is that in a finite length of time it can carry out only a finite number of commands. This is a physical limitation which we humans share with all devices that we can build. It is also a limitation on any being subject to the physical laws of our universe, so any speculation about a being not subject to this limitation belongs to theology rather than science. This result and those related to it would not appear to limit the powers of a being not subject to this physical limitation, and so these results should not offend anybody because of their religion: the limitations apply to us humans and the devices we can build, not to God.

2 A Formal System for Elementary Arithmetic

This is a specification of classical elementary arithmetic. It is an example of the kind of system to which Gödel's Incompleteness Theorem applies.

Definition 1 We assume that we are given infinitely many *variables*: $x_0, x_1, x_2, \ldots$. We will denote these variables by x, y, z, etc. *Terms* are then defined as follows:

1. Every variable is a term.
2. The constant 0 is a term.
3. If s is a term, so is s'.
4. If s and t are terms, so are $s + t$ and st.

We will use 'm^*' as an abbreviation for $0\overbrace{'\cdots'}^{m}$, the term representing the number m. Thus, t' represents the number that is one greater than the number represented by t.

Definition 2 *Formulas* are defined as follows:

1. If s and t are terms, then $s = t$ is a formula.
2. If A is a formula, then so is $\neg A$.
3. If A and B are formulas, then so is $A \supset B$.
4. If A is a formula and x is a variable, then $(\forall x)A$ is a formula.

We will use '$A \wedge B$' as an abbreviation for $\neg(A \supset \neg B)$, '$A \vee B$' for $\neg A \supset B$, and '$(\exists x)A$' for $\neg(\forall x)\neg A$.

Definition 3 An occurrence of a variable x in a formula A is *bound* if it occurs in a subformula of the form $(\forall x)B$; otherwise the occurrence is *free*.

Thus, for example, in $(\forall x)(x = y)$, x is bound and y is free.

Substitution of a term t for a variable x in a formula $A(x)$ is defined so that if a variable occurring free in t would be bound after the substitution, then the bound variable is changed first to prevent this. Thus, for example, if t is x', then substituting t for y in $(\forall x)(x = y)$ results in $(\forall z)(z = x')$ and not in $(\forall x)(x = x')$.

Logic Axiom Schemes If A, B, and C are any formulas, if x is any variable, if t is any term, if $A(x)$ is any formula, and if $A(t)$ is the result of substituting t for x in $A(x)$, then the following are axioms:

1. $A \supset (B \supset A)$.
2. $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$.
3. $(\neg A \supset \neg B) \supset (B \supset A)$.
4. $(\forall x)A(x) \supset A(t)$.
5. $C \supset (\forall x)C$ if x does not occur free in C.
6. $(\forall x)(A \supset B) \supset ((\forall x)A \supset (\forall x)B)$.
7. $(\forall x)(x = x)$.
8. $(\forall x)(\forall y)(x = y \supset (A(x) \supset A(y)))$.

Non-logical axioms

1. $(\forall x)(\forall y)(x' = y' \supset x = y)$.
2. $(\forall x)\neg(x' = 0)$.
3. $(\forall x)(x + 0 = x)$.
4. $(\forall x)(\forall y)(x + y' = (x + y)')$.
5. $(\forall x)(x0 = 0)$.
6. $(\forall x)(\forall y)(xy' = xy + x)$.
7. If $A(x)$ is any formula and x any variable, then $A(0) \wedge (\forall x)(A(x) \supset A(x')) \supset (\forall x)A(x)$ is a non-logical axiom.

Rules of inference

1. From $A \supset B$ and A to deduce B.
2. From $A(x)$, where x is a variable which does not occur free in any assumption on which $A(x)$ depends, to deduce $(\forall x)A(x)$.

Definition 4 A *deduction* from a set Γ of assumptions is a sequence of formulas such that each formula is an axiom or is a formula in Γ or else is derived from one or more previous formulas in the sequence by one of the rules of inference.

We write '$\Gamma \vdash A$' to mean that there is a deduction from the set Γ of assumptions whose conclusion is A.

We write '$\vdash$ A' to mean that $\Gamma \vdash A$ and Γ is the empty set.

The following theorem can be proved about this system:

Theorem 2 *For every partial recursive function $f(x_1, \ldots, x_n)$ there is a formula $A(x_1, \ldots, x_n, y)$ such that $f(m_1, \ldots, m_n) = k$ if and only if $\vdash A(m_1^*, \ldots, m_n^*, k^*)$.*

3 Gödel's Incompleteness Theorem and Its Proof

Gödel's Incompleteness Theorem says that in any formal system which incudes arithmetic that is strong enough to be "interesting," there is a formula which, while true, cannot be proved. More precisely, it says this of any formal system in which all partial recursive functions can be represented in the sense of Theorem 1 of §2.

To get a hint of the proof,[3] imagine that we have the kind of idealized computer described in §1. Anybody with experience using a computer knows that the numerical coding allows the computer to be programmed to determine whether it is reading a single character, a string of characters (a word), a sequence of words, etc. Furthermore, anybody familiar with a programming language or a spreadsheet knows that the computer can be programmed to go back and forth between the character and its number.

Using the definitions in §2, our idealized computer can be programmed to behave rather like a word-processor: it can be programmed to determine whether or not a given number is the code of a variable, the constant, a term, or a formula. It can be programmed to go back and forth between a number m and the term m^* that represents it. It can be programmed to determine whether or not a number codes a variable which occurs free or bound in the formula coded by another number. It can be programmed to determine the number of the formula resulting from the formula corresponding to a given code by the substitution of the term corresponding to a second code for a variable corresponding to a third code. This allows it to be programmed to determine whether a given number codes a logical or non-logical axiom. And it can be programmed to determine whether one number which codes a formula in a sequence of formulas coded by a second number follows from formulas that occur earlier in the sequence by one of the rules. Thus, the computer can be programmed to determine whether a number codes a valid deduction.

By these results and Theorem 1 of §2, there is a formula $G(x, y)$ with two free variables with the property that

$$\vdash G(m^*, n^*) \Longleftrightarrow \begin{array}{l} m \text{ is the number of a formula } A(x) \text{ with one free variable} \\ \text{and } n \text{ is the number of a proof of } A(m^*) \text{ (which results} \\ \text{from } A(x) \text{ by substituting } m^* \text{ for } x) \end{array}$$

[3] A complete proof, which would be too technical for this paper, can be found in [12].

Now let $A(x)$ be the formula $(\forall y)\neg G(x,y)$ (which has one free variable, x). Let g be the number which codes this formula. Then

$$\vdash G(g^*,n^*) \iff g \text{ is the number of a formula } (\forall y)\neg G(x,y) \text{ with one free variable and } n \text{ is the number of a proof of } (\forall y)\neg G(g^*,y)$$

But $\vdash (\forall y)\neg G(g^*,y)$ implies $\vdash \neg G(g^*,n^*)$ for any n. Hence, if the system is consistent,[4] then

$$\nvdash (\forall y)\neg G(g^*,y).$$

But

$$\vdash (\forall y)\neg G(g^*,y) \iff \text{every number is not the number of a proof of } (\forall y)\neg G(g^*,y)$$

or, equivalently,

$$\vdash (\forall y)\neg G(g^*,y) \iff (\forall y)\neg G(g^*,y) \text{ is unprovable}$$

It follows that $(\forall y)\neg G(g^*,y)$ is true, and so it is both true and unprovable. □

One way of looking at Gödel's Incompleteness Theorem is that it says that for any theory strong enough to be "interesting," it is impossible to give any sound and complete axiomatization. Here, an axiomatization does not necessarily have to be finite; it is sufficient that it give an effectively computable test for correctness of any supposed proof. Hence, what Gödel's Incompleteness Theorem really says is that there is no sound and complete specification of the truth conditions of the theory. This applies, in particular, to the theory of elementary arithmetic and to any theory containing it.

On the other hand, just because there is no sound and complete specification of the truth conditions of elementary arithmetic does not mean that we should reject elementary arithmetic or claim that it is impossible to distinguish truth from falsehood in it. Indeed, in order to make sense of the proof of Gödel's Incompleteness Theorem, we need to take elementary arithmetic as a meaningful theory of which we can make sense.

Remark 2 The conclusions of Remark 1 apply to Gödel's Incompleteness Theorem as well as to the undecidability of the halting problem. Indeed, they apply to all the discussions of this paper.

[4]In most systems of formal logic, including the example of §2, a contradiction implies any formula. Hence, in most inconsistent systems any formula can be proved. It is standard practice in science to replace immediately any theory found to be inconsistent, so it makes sense to assume that we are dealing with consistent theories.

4 Implications for Informal Theories

Gödel's Incompleteness Theorem and related results apply directly only to theories that are strictly formalized. In other words, they apply only to theories in which there is a strictly mechanical procedure that could be calculated by our idealized computer that can determine whether a proposed deduction is valid. Most deductive theories are not presented in this form, and so these results do not apply directly to them. However, I think there are implications of these results for such theories.

If an argument is presented in a theory that is not completely formalized but *could* be completely formalized, then the results obviously apply. This would appear to be the case for most theories in mathematics and sciences such as physics that are heavily mathematical, for in these fields it appears that a process of making explicit assumptions that have previously been implicit will, if carried out long enough, lead to a structly formal theory with the same provable results.

However, for fields outside of the mathematical sciences, it is not always clear that theories can be completely formalized in this sense. For such theories, we have no basis for concluding that a result like Gödel's Incompleteness Theorem applies directly. Nevertheless, we cannot ignore the possibility that such a result might apply, for unless there is a disproof, the lack of a proof does not automatically imply that a result is false.

Thus, even if a theory is not compeletely formalized and it is not clear that it can ever be completely formalized, the possibility that incompleteness and related results apply to it cannot be simply ignored. We cannot assume the completeness of any theory without some evidence for it.

Note that the conclusions of Remarks 1 and 2 apply here, so that characterization means characterization by human means (or institutions). It does not really make sense to talk about characterizations without considering the agent doing the characterizing.

5 Applications to Philosophy

I propose here to consider two potential applications of these ideas to philosophy: the philosophy of science and ethics.

5.1 The Philosophy of Science

Here incompleteness would imply that there is no set of rules which can completely characterize the scientific method. I have already suggested [10] that this kind of incompleteness may explain why Karl Popper should not have reacted as negatively as he did [9] to the suggestion by Thomas Kuhn [7] that different scientific paradigms are incompatible. I also suggested there that it may explain why Feyerabend [3] is right to say that the scientific method cannot be completely characterized by a set of rules, but he is wrong to conclude from this fact that there is no such thing as a scientific aproach which is different from other approaches.

5.2 Ethics

Many arguments about ethics take the form of deductive arguments from rules (such as the ten commandments) or arguments about which rules define right and wrong. But suppose that no set of rules can completely characterize the notion of right and wrong. This may imply that we need to think differently about the nature of right and wrong than we have in the past.

As an example, consider the Golden Rule: "Do unto others as you would have others do unto you." This is the well-known Christian form of this rule,[5] but many other religions seem to have a similar idea. In Judaism, for example, there is the well-known saying attributed to Hillel,[6] "Do not do to others what you would not have others do to you. That is the essence of the Law. All the rest is commentary."[7] The same idea occurs in many other influential religions.[8]

[5] It can also be found in Matthew 7:1.

[6] A noted Jewish rabbi who died in the year 10 of the common era. When he said this he was commenting on the Mosaic Law, which is considered by Jews to be the foundation of Judaism.

[7] This can be found in Talmud, Shabbat 3id,

[8] For example, consider the following quotations, which are based on information from `http://www.teaching-values.com/goldenrule.html`: 1) Islam, "No one of you is a believer until he desires for his brother that which he desires for himself." (Source: Sunnah); 2) Bhuddism, "Hurt not others in ways that you yourself would find hurtful." (Source: Udana-Varga 5,1); 3) Hinduism, "This is the sum of duty; do naught onto others that you would not have them do unto you." (Source: Mahabharata 5,1517); 4) Confucianism, "Do not do to others what you would not like yourself. Then there will be no resentment against you, either in the family or in the state." (Source: Analects 12:2); 5) Zoroastrianism, "That nature alone is good which refrains from doing another whatsoever is not good

If the above quotations are taken literally as rules, then there may seem to be differences between them. But there does seem to be a sense in which they all express the idea that it is important to imagine oneself in the situation of others in making ethical decisions. Furthermore, applying the Christian form of this rule in a legalistic way can lead to strange conclusions. For example, I happen to be very fond of the music of Mozart. Suppose I conclude from this that I would be happy to have Mozart's music played over loudspeakers that I can hear all the time.[9] If I apply the Golden Rule in a legalistic way, I could conclude that, as a way of treating others the way I would want to be treated, I should support having Mozart's music played over loudspeakers everywhere all the time. But this is clearly not an appropriate conclusion: some people do not *like* Mozart's music.[10]

Does this not imply that treating the Golden Rule legalistically in this way is a mistake? Should we not instead treat it as an indication of the idea that we should always be trying to put ourselves in others' shoes when making ethical decisions? And should we not also conclude that there is no set of rules that we can use to completely and correctly characterize how we should do this?

It seems clear to me that we cannot do completely without rules for making ethical or moral decisions: life simply does not give us enough time to go back to first principles every time we have to make a decision with ethical or moral consequences. But it may be important to realize that these rules are only guidelines, and in difficult cases they may not be enough to decide what is right and what is wrong.

As a practical matter, I strongly suspect that people who are known for making important ethical or moral decisions under difficult circumstances are motivated mostly by their identification with others. Take, for example, the case of gentiles living under Nazi occupation during World War II who hid Jews from the Gestapo. Did they do this because of a deductive argument about what is right and wrong, or did they do so for emotional reasons? It

for itself." (Source: Dadisten-I-dinik, 94,5); 6) Taoism, "Regard your neighbors gain as your gain, and your neighbors loss as your own loss." (Source: Tai Shang Kan Yin Pien).

[9]Obviously, I am ignoring the fact that if I constantly heard Mozart's music all the time for long enough I would become thoroughly sick of it, but let us suppose for the sake of argument that I am ignoring this problem.

[10]If I correctly understand what American conservative commentator William F. Buckley Jr. has said about Bach, there are Bach fans to whom the music of Mozart sounds too modern.

is worth noting that when Jan Karski, a member of the Polish underground who was being prepared by the underground for a trip to the West in late 1942 to explain the situation in Poland, was shown the Warsaw Ghetto, he had a very emotional reaction, which he describes movingly in [5, Chapter 29]. This reaction was surely not the result of a deductive argument. I suspect that its basis was his identification with the Jews involved as human beings essentially like him. In other words, I suspect that he reacted this way because he could easily imagine being in the place of those Jews.

Another potential example: many first nations in Canada have a tradition of dealing with wrongdoing by means of "healing circles," in which the accused and his/her victims are brought together to discuss the situation and decide on what action to take. Do these healing circles achieve their good effects by inducing the wrongdoer to identify with the victims of his/her actions?

6 Applications to Law

In a letter written in 1981 [8] and widely quoted in 1990 while his confirmation to the U. S. Supreme Court was being considered, Justice David H. Souter, writing for a group of judges, objected to a provision in a proposed state law requiring a judge to determine if an abortion should be performed on a minor when parental consent could not be obtained:

> ... The court's concern is directed ... to that provision of the bill that would require a justice of the Superior Court to authorize the preformance of an abortion upon ... a minor when there is no parental consent, if the justice determines "that the performance of an abortion would be in ... [the] best intersts" of a minor who is "not mature."
>
> The members of the court find two fundamental problems inherent in this provision. First, it would express a decision by society, speaking through the Legislature, to leave it to individual justices of this court to make fundamental moral decisions about the interests of other people without any standards to guide the individual judges. Judges are professionally qualified to apply rules and stated norms, but the provision in question would enact no rule to be applied and would express no norm. In the place of a rule or a norm there would be left only the individual

> judge's principles and predilections. As carefully considered as these might be, they would still be those of only one individual, not those of society. Much criticism of the role of the judiciary in this country has characterized judicial activity in the application of constitutional standards as no more than the imposition of individual judges' views in the guise of applying constitutional terms of great generality. The provision that I have quoted from the present bill would force the Superior Court to engage in just such acts of unfettered personal choice.
>
> The court's second concern is with the necessarily moral character of such choice and the resulting disparity of responses to requests that judicial discretion be exercised. As you would expect, there are some judges who believe abortion under the circumstances contemplated by the bill is morally wrong, who could not in conscience issue an order requiring an abortion to be performed. There are others who believe that what may be thought to be in the "best interests" of the pregnant minor is itself just as necessarily a moral as a social question, upon which a judge may not morally speak for another human being, whatever may be that judge's own personal opinion about the morality of abortion. Judges in each such category would be obligated to indicate that they could not exercise their power in favor of authorizing abortions to be performed on immature pregnant minors. The inevitable result would be required shopping for judges who would entertain such cases. In other words, a principled and consistent application of the quoted provision would be impossible.

This argument against requiring judges to make decisions in the absence of rules or norms to guide them could just as well be made against rules or norms that did not provide adequate guidance to judges; i.e., rules or norms that are not *complete* in an important legal sense. In fact, given the remarks of Judge Souter about the problem of consistency in the application of the law, it is reasonable to suggest that serious incompleteness in the rules and norms provided to judges by a law may cause the most conscientious judge, feeling forced to make a decision despite the incompleteness, to be unable able to avoid inconsistency of application of the law. It seems to me that this might make the law unconstitutional on the grounds that the resulting inconsistency in its application would constitute a violation of the equal protection clause of the Fourteenth Amendment of the United States Constitution and of the Charter of Rights of the Canadian Constitution.

Even if, as R. P. Kerans argued in [6], judges do not simply apply rules to the facts of cases, judges still need some information in the form of rules or norms to guide them so that there is consistency in the application of the law. For this information, it is still reasonable to consider the question of completeness.

For a particular law, it would presumably be easy to determine whether or not its rules and norms are sufficiently complete to pass a constitutional test by examining the record of decisions made under that law or similar laws, particularly for borderline cases. In fact, is this not already a part of our legal system in the sense that courts have in the past ruled laws unconstitutional on grounds of vagueness? On the other hand, it is not at all clear how it might be possible to prove that for a given subject, any set of proposed rules and norms is necessarily incomplete or incorrect.[11] Yet, the above considerations suggest that this possibility cannot be ruled out without some evidence that there is some set of rules and norms which is both complete and correct.

Suppose a legislature passes a law on a certain subject when many attempts to pass a law on that subject have been ruled unconstitutional in the past. Suppose individuals are charged under this new law. Even if this law is eventually ruled unconstitutional, the indivuals charged will have been put through a long, arduous, and expensive legal process before the final ruling on the constitutionality of the law is obtained. In such a situation, might there not be a reasonable presumption that the long history of rulings that laws on the subject are unconstitutional are an indication that it is not possible to pass a legally complete law on this subject, and might that not raise a presumption that nobody should be charged under a new attempt at such a law until its constitutionality is tested in some way? This would seem to suggest some kind of constitutionally based right not to be prosecuted under certain kinds of statutes until their constitutionality is tested. How would it be possible to incorporate such a right in the legal system?

[11] Here, "correct" means adequately characterizing the subject at hand, so that the result of applying the rules and norms will lead to results consistent with the intentions of those proposing them. But in a legal system like that of the United States or Canada, it should also include being consistent with the constitution.

7 Conclusion

In this paper I have begun an inquiry as to whether or not Gödel's Incompleteness Theorem and results related to it apply to areas that are not strictly formalized, in particular to areas of philosophy and the law. My purpose was to ask questions, not give answers. If this work inspires more inquiry into this subject, I will have achieved my purpose.

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