Euclidean Geometry before non-Euclidean Geometry[1]

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In [3], in my argument for the primacy of Euclidean geometry on the basis of rigid motions and the existence of similar but non-congruent triangles, I wrote the following:

A: "The mobility of rigid objects is now recognized as one of the things every normal human child learns in infancy, and this learning appears to be part of our biological progaramming."

B. "… we are all used to thinking in terms of exact scale models, and this is true in every human culture of which I have ever heard."

Marcia Ascher (in private correspondence) suggested that these remarks are ethnocentric.

I think I can justify the basic argument I was trying to make, but my argument appears to have been too simple. The purpose of this paper is to clear up this point. As an illustration of the difficulties, consider the following characterization of Western geometry and the Navajo conception of space from [2, pp. 157–163]:

1. **The Western Case.** I see three main points in the description of Western psychological development of formal and mathematical thought that are developing in a mutually exclusive way in Navajo thought:

   (a) *The Specific Hierarchy.* In his explanation of the child's construction of space, Piaget (with others like Bruner, Goodnow, etc.) gives a detailed analysis of the way more sophisticated notions are linearly deduced or construed in a systematic one-to-one progression from notions acquired earlier. … The notion of "distinctness" is built upon (or deduced from) that of "neighborhood." The notion of "order" is built upon those of "neighborhood" and "distinctness." The notion of "border" is built upon that of "order," and so on until all more sophisticated notions (the projective geometric and Euclidean notions) are integrated in the total conception by similar, quite linear and systematically progressing procedures.

   …

   One fundamental feature of the model of the Western development can thus be summarized as follows: it is a hierarchical progression in the

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sense that each "higher" (more complex) notion necessarily and exclusively implies "lower" (less sophisticated and earlier acquired) notions as constituents.

...

   (b) *The Part/Whole Distinction is Omnipresent.* A certain atomism is obviously characteristic of the current Western style of thought. The "objectification" of the environment is taken for granted in the sciences (situations, static entities can be abstracted from their environment for a certain time) and in school instruction (we study particular animals, places, objects in themselves). ...

   (c) *The Static World.* A very general and possibly somewhat metaphysical sounding characteristic of Western knowledge, as it is practiced and taught, is the static interpretation: The outside world is primarily interpreted as a composition of situations, objects, transitions between situations, and so forth, and not as a composite of processes and actions. ...

2. **The Navajo Case.**

   (a) *The Specific Hierarchy.* In contrast to the neatly regular hierarchical structure and development that were shown in the Western space conceptualization, Navajo space (according to our semantic analysis) appears to be founded on at least three equally important "basic" notions (movement, volumeness/planeness, dimensions). All three are topological in character. Moerover, none of them are really "primitive" in the sense that the (Piagetian) Western notions are: They are clearly composites themselves, and have spatial notions as their constituents; they codetermine themselves. Hence, they exhibit a certain circularity. Finally, they cannot, in any strict sense, be said to be the sole "basic constituents" since they have this status by virtue of the fact that they were selected as the notions involved in the constitution of most other notions. ...

   (b) *The Part/While Distinction is Secondary.* The part/whole distinctions that proved so important in Western thought and knowledge systems play a minor role in Navajo knowledge. Navajos tend to speak of the world in terms of process, event, fluxes, rather than parts and wholes or clearly distinguishable static entities. The emphasis is on continuous changes rather than on atomistic structure. ...

   (c) *The Dynamic World.* In earlier swections it was pointed out that the Navajo world view stresses the dynamic rather than the static aspects of reality. ...

This shows that my statements A and B above are too simple.

However, the basis of non-Euclidean geometry is the same in terms of the above classification as is that of Euclidean geometry; non-Euclidean geometry is based just as much on the specific hierarchy, the part/whole distinction, and the static world as is Euclidean geometry. So we can see immediately that the Navajo are no more likely to arrive at non-Euclidean geometry than they are at Euclidean geometry. The same would hold for other cultures to which these Western notions are foreign.

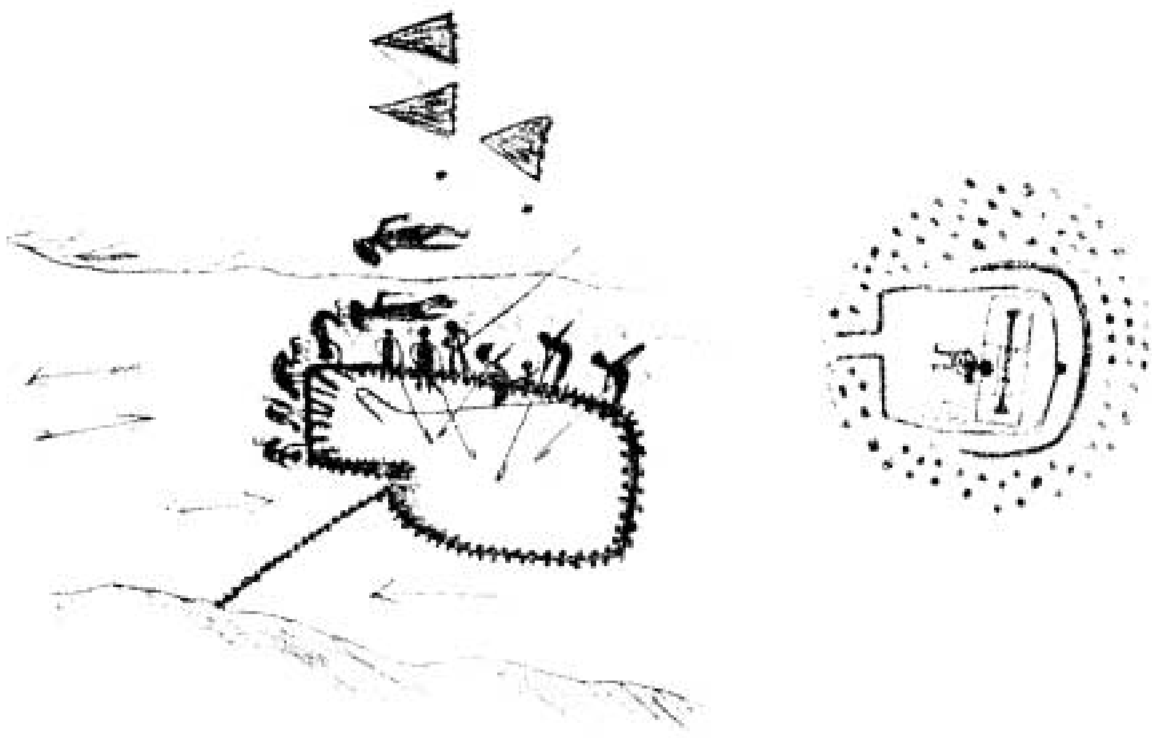
Furthermore, we can talk about observers from a different culture from any culture being studied; indeed, Pinxten and colleagues are just such observers. And it makes sense to talk about the way such observers might analyze the culture being studied. Let us assume that all the observers are from our culture with our Western notions as the basis of their thinking.

With regard to A: Consider any culture in which things are made which require parts to be fitted together. The fitting of such parts (which we may assume to be those the observers would regard as rigid) requires some equivalent of measurement, and measurements (and fits of parts) will not change as the objects are moved. To our observers, this would imply the existence of rigid motions, even if the culture being observed did not have the concept internally. If an individual from that culture managed to internalize enough basic ideas of the observers, that individual would have to accept that rigid motions exist, even if the thought would seem somewhat unnatural.

With regard to B: It is certainly true that global scale models are not universal. But many cultures that do not have global scale models do have representations with local scale representations: there are parts of these representations that suggest what they represent by their shape. Consider the following two examples:

1. The representing of musk-ox hunting in the following figure is not globally to scale. However, the individual figures indicate what they represent (men or animals) by their shape: each figure *has the same shape* as the animal or man that it represents. The figure is [1, Figure 5.2]; the caption reads "Musk-ox hunting on North Somerset Island. Drawn by Itqilik."

2. The catching of fish in the following figure is, again, not a global scale representation in our Western sense. But again, different parts of this figure indicate what they represent by their shape. The figure is [1, Figure 5.3]; the caption reads "Catching fish in a *saputit* and in a *kapisilingniarfit*. Drawn by Qavdlunâq."



This use of sameness of shape (as analyzed by the observers) is common to many cultures, perhaps all that have pictorial representations. To an outside observer of these culture, this is enough to imply the existence of triangles that are similar but not congruent. Since only one pair of similar but not congruent triangles is enough to imply Wallis' theorem, this is sufficient for B.

Thus, while the notions of Euclidean geometry may be foreign to the culture being studied, the observers will be led to find Euclidean the most natural from their point of view. Furthermore, I think this also shows that *if a culture takes up the study of geometry in a systemmatic way based on the Western (Greek) concepts, it will reach Euclidean geometry before non-Euclidean geometry.*

References

[1] Marcia Ascher. *Ethnomathematics: A Multicultural View of Mathematical Ideas.* Brooks/Cole, Pacific Grove, California, 1991.

[2] Rik Pinxten, Ingrid van Dooren, and Frank Harvey. *Anthropology of Space: Explorations into the Natural Philosophy and Semantics of the Navajo.* University of Philaphadelphia Press, 1983.

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